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Geometrijski modeli votkovne zanke: ohlapna, normalna in zbita struktura

Pregledni znanstveni članek

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Geometrical Models of Weft Knitted Loop: Open, Normal and Compact Structure

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Abstract

In the first half of the 20th century, scientists analyzed the single knitted structure through the studies of the knitted structure basic element, i.e. the knitted loop. By means of loop models, they tried to define the relationships among yarn parameters, knitted fabric parameters and knitting process parameters. The first geometrical loop models played an important role in the control of knitted fabric dimensions and mass per unit area, while the contemporary ones have been designed for graphic simulations and knitted structure planning. The real knitted fabrics are complex materials which do not meet the applied idealized presumptions, i.e. structural homology, non-compressibility and simple geometry. Recently, the studies of the knitted loop geometry have been a research subject once again, e.g. for the demands of the hosiery sphere, computer simulations of the knitted fabric appearance, planning of knitted composites and knitted structures with unconventional properties. New expertise, modern testing and measuring techniques, e.g. electron microscopy and computer picture analysis enable the specification of the loop shape and size in dependence on the geometrical parameters of the

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Izvleček

Znanstveniki so že v prvi polovici 20. stoletja na podlagi študija temeljne celice pletiva – zanke analizirali levo-desno pleteno strukturo in poskušali z modeli opisati razmerja med parametri preje in pletiva ter procesnimi parametri pletenja. Prvi geometrijski modeli so odigrali pomembno vlogo pri nadzorovanju dimenzij in ploščinske mase pletiva, sodobnejši pa so namenjeni grafičnim simulacijam in projektiranju pletene strukture. Resnična pletiva so kompleksni materiali, ki ne ustrezajo uporabljenim idealiziranim predpostavkam: strukturna homogenost, nestisljivost in preprosta geometrija. Študij geometrije zanke je v zadnjem času ponovno predmet raziskav, npr. za potrebe nogavičarstva, računalniških simulacij videza pletiva, projektiranja kompozitnih tekstilij ter pletenih struktur z nekonvencionalnimi lastnostmi. Nova spoznanja ter sodobne preskuševalne in merilne tehnike, npr. elektronska mikroskopija in računalniška slikovna analiza, omogočajo opis oblike in dolžine zanke v odvisnosti od geometrijskih parametrov zanke. Opisani in analizirani so najznačilnejši in najpomembnejši konstrukcijski geometrijski modeli zanke votkovnega pletiva.

Ključne besede: model zanke, zanka, pletena struktura, dolžina zanke

Uvod

Veliko znanstvenikov je študiralo pleteno, predvsem levo-desno strukturo in njeno temeljno enoto, zanko. Njihove raziskave je mogoče razdeliti v štiri glavne skupine:

a) geometrijski modeli zanke: Peirce [1], Leaf in Glaskin [2], Munden [3], Vekassy [4], Suh [5], Dalidovič [6, 7], Korlinski [8], Mo-

loop. In the present study, the most distinctive and important constructional geometrical weft loop models are presented and analyzed.

Keywords: loop model, loop, knitted structure, loop length

1 Introduction

Many scientists have been studying knitted fabrics, mainly the single structure, and its basic element – the knitted loop. Their investigations can be subdivided into four main groups:

a. geometrical loop models: Peirce (1), Leaf & Glaskin (2), Munden (3), Vekassy (4), Suh (5), Dalidovich (6,7), Korlinski (8), Morooka & Matsumoto & Morooka (9),

b. mechanical loop models: Leaf (10), Nutting & Leaf (11), Postle (12), Whitney & Epting (13), Popper (14), Postle & Munden (15,16), Shanahan & Postle (17), Hepworth & Hepworth & Leaf (18), MacRory et al (19), Hepworth & Leaf (20), Hepworth (21), Wu & Hamada & Maekawa (22),

c. energy loop models: Glaskin & Leaf (23), Postle & Carnaby & deJong (24), Kawabata (25),

d. experimental research – testing of the loop models adequacy for the real knitted fabrics: Doyle (26), Fletcher & Roberts (27-32), Shinn (33), Munden & Fletcher (34), Leaf (35), Lau & Dias (36), and many others.

As early as in the first half of the 20th century, scientists tried to define the relationships among yarn parameters, knitted fabric parameters and knitting process parameters. In the second half of the century, they started mathematically describing the changes in loop dimensions and shape during uniaxial and biaxial stresses. The primary geometrical loop models were relatively simple, yet they only corresponded to a limited extent to the yarns and the knitted structures applied at that time. With the new information technology, testing equipment and fibre development, they were first replaced with new, more complex and comprehensive mechanical loop models, and afterwards with energy models.

The first geometrical loop models played an important role in the control of knitted fabric dimensions and mass per unit area, while

rooka in Matsumoto in Morooka [9],

b) mehanski modeli zanke: Leaf [10], Nutting in Leaf [11], Postle [12], Whitney in Epting [13], Popper [14], Postle in Munden [15, 16], Shanahan in Postle [17], Hepworth in Hepworth in Leaf [18], MacRory et al. [19], Hepworth in Leaf [20], Hepworth [21], Wu in Hamada in Maekawa [22],

c) energijski modeli zanke: Glaskin in Leaf [23], Postle in Carnaby in deJong [24], Kawabata [25],

d) eksperimentalno delo – preskušanje ustreznosti modelov za dejansko pletivo: Doyle [26], Fletcher in Roberts [27–32], Shinn [33], Munden in Fletcher [34], Leaf [35], Lau in Dias [36] in mnogi drugi.

Že v prvi polovici 20. stoletja so poskušali z modeli opisati razmerja med parametri preje in pletiva ter procesnimi parametri pletenja. V drugi polovici stoletja so se lotili matematičnega opisa sprememb dimenzij in oblike zanke med enoosnim in dvoosnim obremenjevanjem. Prvotne, razmeroma preproste geometrijske modele zanke, ki so že ob nastanku le v omejenem obsegu ustrezali takratnim uporabljenim prejam in strukturam pletiva, so z razvojem informacijske in preskuševalne opreme ter novih vlaken in prej zamenjali novi mehanski, bolj zapleteni in obsežnejši modeli ter nato energijski modeli.

Prvotni geometrijski modeli so odigrali pomembno vlogo pri nadzorovanju dimenzij in ploščinske mase pletiva, zadnji pa so namenjeni grafičnim simulacijam pletene strukture. Študij geometrije zanke je v zadnjem času ponovno predmet raziskav, npr. za potrebe nogavičarstva, računalniških simulacij videza pletiva, projektiranja kompozitnih tekstilij ter pletenih struktur z nekonvencionalnimi lastnostmi. Podprt z novimi izkušnjami ter preskuševalnimi in merilnimi tehnikami, npr. elektronsko mikroskopijo in računalniško slikovno analizo, omogoča opis oblike in dolžine zanke v odvisnosti od geometrijskih parametrov zanke.

2 Geometrijski modeli zanke

2.1 Peirceov model zanke

Peirce [1] je pri študiju zanke predpostavil, da ima pletivo normalno strukturo, tj. da se sosednje niti v pletivu stikajo (slika 1). Pro-

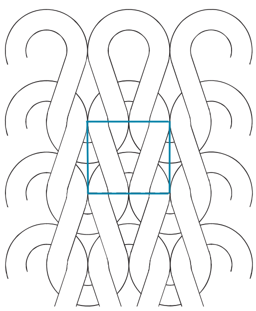

Figure 1: Peirce's loop model (1) – normal structure

the contemporary ones have been designed for graphic simulations and knitted structure planning. Recently, the studies of the knitted loop geometry have been a research subject once again, e.g. for the demands of the hosiery sphere, computer simulations of the knitted fabric appearance, planning of knitted composites and knitted structures with unconventional properties. Supported by new expertise, modern testing and measuring techniques, e.g. electron microscopy and computer picture analysis, they enable the specification of the loop shape and size in dependence on the geometrical parameters of the loop.

2 Geometrical loop model

2.1 Peirce's loop model

In his study, Peirce (1) presumed that a knitted structure is normal when the adjacent yarns within the knitted fabric are in contact (cf. Figure 1). The projection of the loop onto the fabric plane is composed of the circular needle and sinker arcs connected with straight lines, i.e. loop legs (cf. Figure 2). The loop is three-dimensional, which means that the loop arcs and legs lie on the cylinder surface with the curvature radius R and the axis parallel to the course direction (cf. Figure 3).

If A be loop width, B loop height, ℓ loop length and d_{pr} yarn diameter, we have for the normal knitted structure (cf. Figure 2) (Equation 1 and 2).

Moreover, we have the quarter of the loop length ℓ / 4 (cf. Figure 2) (Equation 3), and therefore (Equation 4).

For the normal structure of the knitted fabric (1), the ratio between the radius R of the cylinder on which the loop lies and the yarn diameter d_{pr} is a constant (cf. Figure 3) (Equation 5).

Taking into account that a real knitted fabric does not necessarily exist as a normal structure, Peirce (1) adjusted his loop model also for open knitted fabrics in which the adjacent yarns are not in contact. In this case, he anticipated loop elongation by inserting straight yarn segments: a segment in the crown of the loop parallel to the course line – ε · d_{pr}, and two straight yarn segments between the arcs and loop legs – ξ · d_{pr}. The coefficient ξ can have a negative value

jekcijo Peirceovega modela zanke v ravnini sestavljajo krožni igelni in platinski loki zanke, ki jih povezujeta ravni črti – kraka zanke (slika 2). Zanka je tridimenzionalna; loki in kraki zanke ležijo na površini valja s polmerom *R* in z osjo, vzporedno z zančnimi vrstami (slika 3).

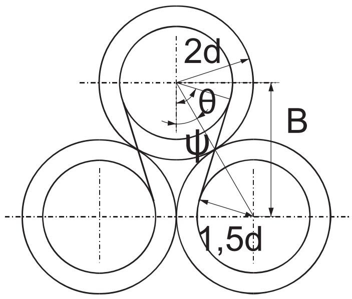

Figure 2: Peirce's loop model (1) – construction of needle and sinker arc, and loop legs (projection onto fabric plane)

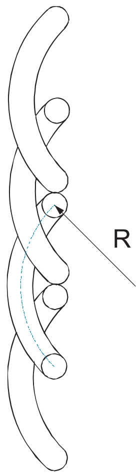

Figure 3: Peirce's loop model (1) – curvature of loop legs

Če je: *A* – širina zanke, *B* – višina zanke, *ℓ* – dolžina zanke in d_{pr} – premer preje, za pletivo normalne strukture velja (slika 2):

$$A = 4\,d_{pr} \tag{1}$$

in

$$B^2 = (4\,d_{pr})^2 - (2\,d_{pr})^2 = 12\,d_{pr} \Rightarrow \mathrm{B} = 2\,d_{pr}\,\sqrt{3} \tag{2}$$

Velja tudi, da je četrtina dolžine zanke *ℓ / 4* (slika 2):

$$\frac{\ell}{4} = \frac{3\,d_{pr}}{2}\,(\pi - \theta) + 2\,d_{pr} \cdot \sin(\theta - \psi) \tag{3}$$

in torej

$$\ell = 16.6628\,d_{pr} \tag{4}$$

up to −0.34. The loop length of the open knitted structure ℓ defined by Peirce (1) is (Equation 6).

Furthermore, Peirce (1) estimated that the knitted structure becomes compact (crammed) when the ratio between the loop length and yarn diameter (linear loop module) attains the value $\ell / d_{pr} = 16$.

2.2 Leaf & Glaskin's loop model

Leaf & Glaskin (2) disproved Peirce's model of the loop shape (1) and demonstrated that it was physically impossible due to the discontinuities in the torsion occurring round the loop. They proposed a new, three-dimensional loop model (2), composed only of circular arcs in the projection onto the fabric plane (cf. Figure 4).

The basic parameters of Leaf & Glaskin's loop model (2) are (cf. Figure 4): d_{pr} for yarn diameter; a_{LG} for the factor of the circular arc diameter ($a_{LG} \cdot d_{pr}$ is the radius of the circular arcs composing the loop in the projection on the fabric plane, i.e. a needle arc and two halves of the sinker arcs); φ for the angle between the distance from the needle loop centre C to the lowest point of the loop O regarding the fabric plane, and the distance from the needle loop centre C to the point Q, where the central line of the needle arc with the centre C joins the central line of the sinker arc with the centre F being the highest point of the loop regarding the fabric plane; h for the distance (height) of the point Q regarding the projection plane of the fabric.

Leaf & Glaskin (2) mathematically proved that in order for the arcs of the adjacent loops (point G in Figure 4) to fit together, restrictions must be imposed on the values a_{LG} and φ: the smallest possible value of the factor of the circular arc diameter $a_{LGmin} = 1.5$. As $a_{LG} \to \infty$, we have $90° < \varphi < 150°$. Their three-dimensional loop model consisted of both loop arcs and loop legs being circular forms exclusively, lying on the cylindrical surfaces. They explained the adequacy of such a loop shape with the fact that the loops are formed during knitting by bending the yarn round the cylindrical knitting elements.

The loop length of their three-dimensional model is equal to four times the portion of the arc OQ (cf. Figure 4), which is a complex mathematical equation. A simplified derivation of

Razmerje med polmerom valja, na katerem leži zanka, R, in premerom preje, $d_{pr,}$ je pri pletivu z normalno strukturo konstanta [1] (slika 3):

$$\frac{R}{d_{pr}} = 4.172 \qquad (5)$$

Glede na to, da realno pletivo nima nujno normalne strukture, je Peirce [1] prilagodil svoj model zanke tudi za ohlapna pletiva, pri katerih se sosednje niti ne stikajo. Pri tem je predvidel podaljšanje zanke za ravne odseke niti: odsek v glavi zanke, vzporeden s smerjo zančne vrste – $\varepsilon \cdot d_{pr}$, ter dva ravna odseka med loki in krakoma zanke – $\xi \cdot d_{pr}$. Koeficient ξ ima lahko negativno vrednost do −0,34. Dolžina zanke ohlapnega pletiva ℓ je po Peirceu [1]:

$$\ell = 2\,A + B + 5.94\,d_{pr} \qquad (6)$$

Peirce [1] je tudi ocenil, da postane struktura pletiva zbita, ko razmerje dolžine zanke in premera preje (dolžinski modul zanke) doseže $\ell / d_{pr} = 16$.

2.2 Leaf-Glaskinov model zanke

Leaf in Glaskin [2] sta ovrgla Peirceov model oblike zanke [1] in dokazala, da je fizikalno nemogoč zaradi nezveznosti torzije vzdolž niti v zanki. Predlagala sta nov prostorski model zanke [2], pri katerem projekcijo na ravnino pletiva sestavljajo le krožni loki (slika 4).

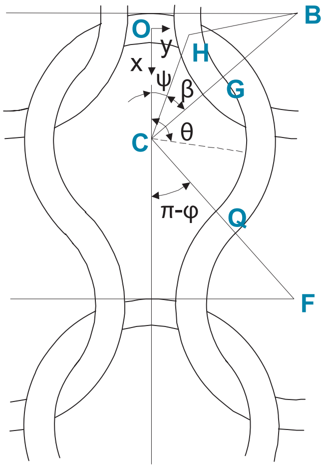

Figure 4: Projection of Leaf & Glaskin's (2) loop model onto fabric plane

Temeljni parametri modela zanke Leaf-Glaskina [2] so (slika 4): d_{pr} – premer preje; a_{LG} – faktor polmera krožnih lokov ($a_{LG} \cdot d_{pr}$ je polmer krožnih lokov, ki v projekciji sestavljajo zanko, tj. igelni lok in polovici platinskega loka); φ – kot med razdaljama od središča C igelne-

the loop length with an error less than 6% is the length of the projection of the loop onto the fabric plane (2) (Equation 7).

2.3 Munden's loop model

In Munden's view (3), the starting point of assessing the relation between the yarn parameters and knitted fabric parameters is the yarn and the fabric history elimination, or the definition of the reference state, namely the relaxed state of the knitted fabric.

Munden's loop model (3) is three-dimensional. The yarn is bent both in the plane of the fabric and in the plane at right angles of the fabric (cf. Figure 5). The interlaced loops form a flat structure. The needle loops and sinker loops are identical with Munden's loop model (cf. Figure 6).

Munden's model is based on the presumption that the loop shape is a geometrical property and is independent from the yarn physical properties or loop length. The model was supported with Leaf's findings (37) that a homogeneous strip of yarn bent into a loop in one plane by bringing its two ends together and parallel, providing the strip is not plastically deformed by bending, will take up a particular configuration which is independent from the physical properties, thickness, or length of the material forming the loop. According to Munden (3), the loop shape is universal if the second presumption holds true, namely that the knitted fabric is in a fully relaxed state or the state of minimum energy. With Munden's model, the interlacing points between the loops occur at the same relative position on the curve, irrespective of the size of the loop. The narrowest point of the loop and the widest point of the loop of the previous course lie on the same axis, parallel to the abscissa (cf. Figure 6).

According to Munden (3), the primary parameter of the knitted fabric is loop length ℓ, dependent only on fabric density, while the ratio between the knitted fabric density and the loop length is independent from the fabric structure, i.e. its openness (3). The loop width (knitted fabric horizontal density) and loop length (knitted fabric vertical density) are directly inversely proportional to the loop length. Munden defined the interdependence of individual loop parameters with the so-called Munden

ga loka zanke do najnižje točke zanke O glede na ravnino pletiva ter od središča C igelnega loka zanke do točke Q, kjer se središčna os igelnega loka s središčem C združi s središčno osjo platinskega loka s središčem F, in ki je hkrati najvišja točka zanke glede na ravnino pletiva; h – razdalja (višina) točke Q od projekcijske ravnine pletiva.

Leaf in Glaskin [2] sta matematično dokazala, da je pogoj za stikanje lokov sosednjih zank (točka G na sliki 4) omejitev vrednosti a_{LG} in φ: najmanjša vrednost faktorja polmera krožnih lokov a_{LGmin} = 1,5. Če je $a_{LG} \rightarrow \infty$, je $90° < \varphi < 150°$. Ustreznost prostorske oblike zanke, pri kateri so tako zančni loki kot kraki krožni odseki, ležeči na površinah valjev, sta utemeljevala z dejstvom, da so zanke med pletenjem oblikovane na valjastih pletilnih elementih.

Dolžina zanke njunega prostorskega modela je štirikratna dolžina loka OQ (slika 4) in zapleten matematični izraz. Poenostavljena izpeljana dolžina zanke z napako, manjšo od 6 %, je dolžina projekcije zanke v ravnini pletiva [2]:

$$\ell = 4\, a_{LG} \cdot \varphi \cdot d_{pr} \tag{7}$$

2.3 Mundenov model zanke

Izhodišče ugotavljanja razmerij med parametri preje in pletiva je po Mundenu [3] izločitev vplivov zgodovine preje in pletiva oz. definiranje referenčnega, tj. relaksiranega stanja pletiva.

Mundenov model zanke [3] je prostorski; preja je upognjena v zanko v ravnini pletiva ter v ravninah, pravokotnih na ravnino pletiva (slika 5). Med seboj povezane zanke oblikujejo plosko strukturo. Igelna in platinska glava sta pri Mundenovem modelu zanke enaki (slika 6).

Mundenov model zanke temelji na predpostavki, da je oblika geometrijska lastnost zanke in je neodvisna od fizikalnih lastnosti preje

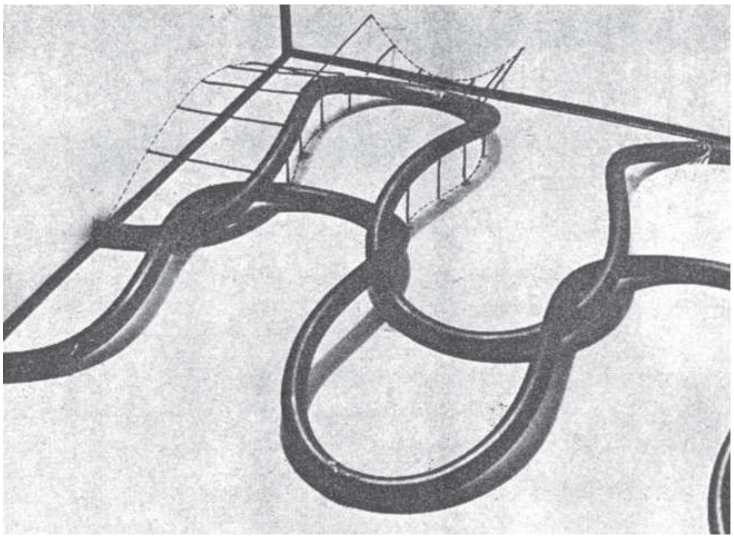

Figure 5: Munden's three-dimensional model (3) of single weft knitted structure and its projections

constants (cf. Equations 8–12); where D is knitted fabric area density, D_v knitted fabric vertical density, D_h knitted fabric horizontal density, ℓ loop length, and K_1, K_2, K_3 and K_4 Munden constants.

2.4 Vekassy's loop model

Vekassy (4) derived his loop model from Dalidovich's model of the non-stressed loop (6) and the results of Doyle's research (26). He presumed that the yarn is entirely even, its cross section circular and its diameter a constant. Vekassy's loop model is three-dimensional. The knitted loop is defined with a space-curve $E_1A_1I_1$, namely a cycloid arising from crossing the cylinder jacket H with the radius R and horizontal axis, with three parallel cylinder jackets F, F_1 and F_2 with equal radii r and vertical axes, and two planes S and S_1, parallel to the axes of the cylinders with the radii r (cf. Figure 7). The loop consists of four equally long parts as $A_1B_1C_1$ (cf. Figure 8).

Vekassy (4) calculated the loop length (Equation 13), with (cf. Figures 7 and 8): ℓ for loop length; R for the radius of the cylinder holding up the loop; r for the radii of the cylinders around which the needle and sinker arcs are bent; a_V for the projection of the half of the loop leg onto the vertical axis in the fabric plane (namely half of the loop height); and b_V for the distance between the projection of the loop arc centre onto the fabric plane, and the projection of the part of the loop bisection and the line holding the loop leg in the same plane.

Vekassy (4) also defined the simplified equation for the loop length of the normal knitted structure in which the needle and sinker arcs are in contact. He presumed that the height of the circular needle arc with the radius R equals yarn diameter. The loop length of the normal structure is (Equatin 14), where ℓ is loop length and d_{pr} yarn diameter.

Moreover, Vekassy (4) anticipated the structure being more compact than the normal structure. He presumed that the needle and sinker arcs are elliptical and that they are in contact in both horizontal and vertical direction. With yarn of the same diameter, the loop width of the compact structure equals the loop width of the normal knitted structure. At the same time, the

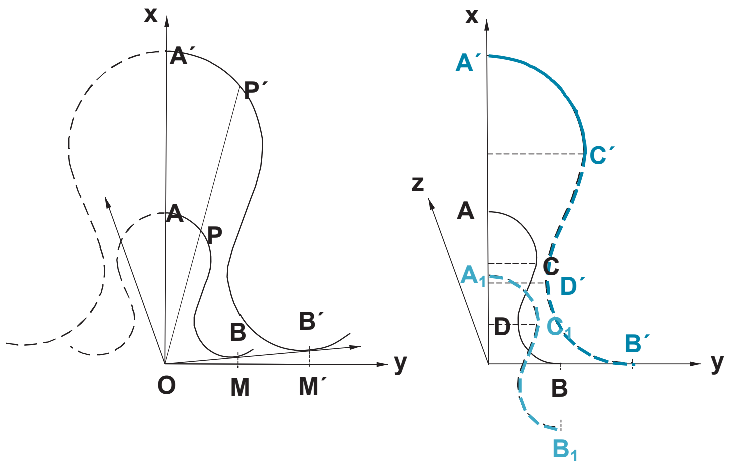

Figure 6: Projection of Munden's loop model (3) onto fabric plane

ali dolžine zanke. Pri tem se opira na ugotovitve Leafa [37], da homogen odsek preje, ki je upognjen tako, da sta konca vzporedna in pri katerem zaradi upogiba ne pride do plastične deformacije, zavzame določeno obliko, neodvisno od fizikalnih lastnosti, debeline ali dolžine materiala, ki oblikuje zanko. Oblika zanke je po Mundenu [3] univerzalna, če velja druga predpostavka, tj. popolnoma relaksirano stanje oz. stanje najmanjše energije pletiva. Pri Mundovem modelu zanke so stične točke sosednjih zank pletiv z različno dolžino zanke na relativno vedno enakem mestu; točka v najožjem delu zanke in točka v najširšem delu predhodne zanke istega zančnega stolpca ležita na isti osi, vzporedni z absciso (slika 6).

Temeljni parameter pletiva po Mundenu [3] je dolžina zanke ℓ, ki je odvisna le od gostote pletiva. Razmerje med gostoto pletiva in dolžino zanke je pri Mundenovem modelu [3] neodvisno od strukture, tj. zbitosti/ohlapnosti pletiva, širina in višina zanke oz. horizontalna in vertikalna gostota pletiva pa sta direktno obratnosorazmerni z dolžino zanke. Munden je odvisnost posameznih parametrov zanke definiral s t. i. Mundenovimi konstantami (enačbe 8–12):

$$D = \frac{K_1}{\ell^2} \quad (8),$$

$$D_V = \frac{K_2}{\ell} \quad (9),$$

$$D_h = \frac{K_3}{\ell} \quad (10),$$

$$K_1 = K_2 \cdot K_3 \quad (11),$$

$$\frac{D_V}{D_h} = \frac{K_2}{K_3} = K_4 \quad (12),$$

loop height of the compact structure is smaller than the height of the normal loop structure (cf. Figure 9).

Vekassy evaluated the value of the half of the small axis of the median of the needle loop ellipse b_1 on the basis of the dimension measurements of the magnified loop pictures: b_1 = 0.9d_{pr}. According to Vekassy, the loop length of the compact knitted structure is (4) (equation 15), where ℓ is loop length and d_{pr} yarn diameter.

2.5 Suh's loop model

In his research, Suh (5) studied the phenomenon of a cotton single knitted fabric shrinking as a result of the yarn swelling during wetting and washing. He explained the shrinking in the longitudinal direction with the change of the loop shape after wetting and drying, and the transversal shrinking with the changed ratio between the horizontal density and yarn diameter.

Suh presumed that the yarn within the loop has a circular cross section and a uniform diameter. The yarns of the adjacent loops are in contact at points M and N. The needle arc MN equals the sinker arc DA with the length $L_{i,p}$ and is the portion of the circle on the chord DA. The distance from the line MN to the centre of the loop tip Q is y (cf. Figures 10 and 11). The loop width is A, loop height B and the loop leg length is L_k. The loop leg lies on the concave plane S with the radius ρ. The loop flexion is defined with the loop height B, the height of the needle or singer arc y and the yarn diameter d_{pr}. From Figure 12, it can be seen that the radius of the loop leg arc ρ as well as the loop flexion decrease with the loop flexion chord B + 2y decrease and/or the yarn diameter d_{pr} increase. Thus, the vertical density of the knitted fabric increases.

The sinker and needle arcs are not semicircles but arcs with the central angle 2φ < 180° (cf. Figure 11). The angle between the tangent of the needle arc and the vertical line α_S equals the angle of the inclination of the loop legs with the length L_k towards the vertical line (cf. Figures 10 and 11). It follows that the needle or sinker arc height $y < r_S$ if r_S is the radius of the needle or sinker arc.

Owing to a simplified loop length calculation, Suh presumed that the length of the three-di-

kjer so: D – ploskovna gostota pletiva, D_v – vertikalna gostota pletiva, D_h – horizontalna gostota pletiva, ℓ – dolžina zanke in K_1, K_2, K_3, K_4 – Mundenove konstante.

2.4 Vekassyjev model zanke

Vekassy [4] je izpeljal svoj model zanke iz modela neobremenjene zanke Dalidoviča [6] in rezultatov raziskav Doyla [26]. Pri

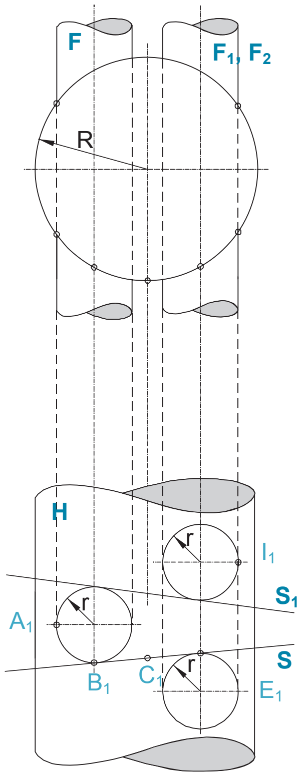

Figure 7: Vekassy's loop model (4) – definition of space-curve

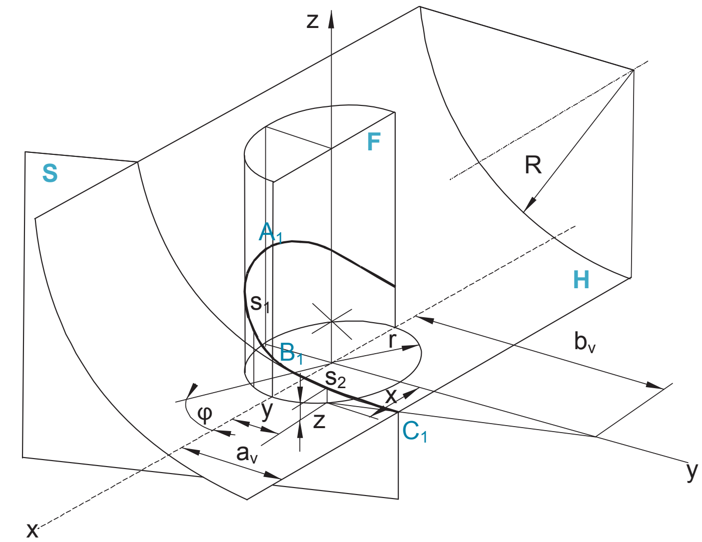

Figure 8: Space-curve $A_1B_1C_1$ of Vekassy's loop model

mensional needle and sinker arcs approximately equals the projection of the arc onto the fabric plane, and the loop leg length L_k equals the length of the arc with the radius ρ, since the loop leg L_k lies on the concave surface S with the radius ρ. From Figure 11, it can be seen that the length of the needle and sinker arc $L_{i,p}$ is (Equation 16).

The loop leg length L_k is (Equation 17).

and the loop length ℓ is (Equation 18), where ℓ is loop length, A loop width, B loop height, y needle or sinker arc height, d_{pr} yarn diameter and ρ loop leg arc radius.

2.6 Dalidovich's loop model

Dalidovich's general loop model (6, 7) is three-dimensional. It presumes that the knitted structure is open, and that the distance between the sinker and the needle arcs of the loop is k (cf. Figure 13).

According to Dalidovich, the loop length is a function of the loop width A, loop height B and yarn diameter d_{pr}. The simplified loop length considering the vertical loop legs with the length equal to the loop height is (cf. Figure 13), (Equation 19).

From Figure 13, it can be seen that (Equation 20 and 21).

Assuming the simplifications that the loop is planar, that the loop legs are parallel to the ordinate and their length equals the loop height B, it follows from the Equations 19–21 (Equation 22).

If the loop is planar and the loop legs lie inclined within the loop plane as shown in Figure 13, it holds (Equation 23).

With the three-dimensional position of the loop legs within the loop, taking into account the simplified folded linear shape of the loop legs, the equation gets the form of (Equation 24), where ℓ is loop length, A loop width, B loop height, D_d the radius of the central arc of the needle/sinker loop, d_{pr} yarn thickness, and k the distance between the needle and sinker arcs of the loop.

In case that the needle and sinker arcs of the loop are horizontally and vertically in contact, namely within the normal knitted structure, k = 0. Therefore, it follows from Equation 20 that $A = 4d_{pr}$.

tem je predvideval, da je preja enakomerna in ima okrogel prerez s konstantnim premerom. Vekassyjev model zanke je prostorski. Zanko definira prostorska krivulja $E_1A_1I_1$, tj. cikloida, ki nastane s presečiščem krive ploskve H, ki je plašč valja s polmerom R in vodoravno osjo, treh krivih ploskev F, F_1 in F_2, ki so plašči vzporednih valjev z enakimi polmeri r in navpičnimi osmi, ter dveh ravnin S in S_1, vzporednih z osmimi valji s polmerom r (slika 7). Zanko sestavljajo štirje enaki deli, kot je $A_1B_1C_1$ (slika 8).

Vekassy [4] je izračunal dolžino zanke:

$$\ell = 4\left[-\frac{r}{6R^2}\left(3\sqrt{2}\,R^2 + 4\sqrt{2}\,r^2 + 6\,r\cdot a_V + 6\sqrt{2}\,a_V^2\right) + \frac{r\left(R^2 - a_V^2 - r^2\right)}{2R^2}\cdot\ln\frac{2-\sqrt{2}}{2+\sqrt{2}} - \frac{r^2\cdot a_V}{R^2}\cdot\ln\frac{1}{2} + \frac{1}{4\,b_V\cdot\sqrt{r^2+b_V^2}}\left(4\,a_V\cdot r^2 + 2\,a_V\cdot b_V^2 - b_V^2\cdot R\cdot\ln\frac{R-a_V}{R+a_V}\right)\right] \quad (13)$$

pri čemer je (sliki 7 in 8): ℓ – dolžina zanke; R – polmer valja, na katerem leži zanka; r – polmeri valjev, okrog katerih se ovijajo igelni lok in platinska loka; a_V – projekcija polovice dolžine zančnega kraka na vzdolžno os zanke v ravnini pletiva (tj. polovica višine zanke) in b_V – razdalja med projekcijo središča igelnega loka v ravnini pletiva ter projekcijo presečišča simetrale zanke in premice, na kateri leži krak zanke v isti ravnini.

Vekassy [4] je podal tudi poenostavljeno enačbo za izračun dolžine zanke normalne strukture pletiva, pri kateri se igelni in platinski loki vzdolžno in prečno stikajo. Predpostavil je, da je višina igelnega loka kroga s polmerom R enaka premeru preje. Dolžina zanke normalne strukture pletiva je:

$$\ell = 17.33\,d_{pr} \quad (14),$$

pri čemer je: ℓ – dolžina zanke in d_{pr} – premer preje.

Vekassy [4] je predvidel tudi strukturo, bolj zbito od normalne. Pri tem je predpostavil, da so igelni in platinski loki eliptične oblike ter se vzdolžno in prečno stikajo. Pri istem premeru preje je širina zanke zbite strukture enaka širini zanke normalne strukture pletiva, višina zanke zbite strukture pa je manjša od višine zanke normalne strukture pletiva (slika 9).

Malo polos središčne igelne elipse b_1 je Vekassy [4] ocenil na podlagi meritev dimenzij povečane slike modela zanke zbite strukture b_1 = 0,9 d_{pr}. Dolžina zanke zbite strukture je po Vekassyju [4]:

$$\ell = 13.3964\,d_{pr} \approx 13.40\,d_{pr} \quad (15),$$

pri čemer je: ℓ – dolžina zanke in d_{pr} – premer preje.

With the normal knitted structure, the relation is (Equation 25), hence, the ratio between the loop height and loop width, or the ratio between the horizontal and vertical density for the geometrical Dalidovich's loop model equals the knitted fabric density coefficient of the normal structure C = 0.866. The loop length of the normal structure is (Equation 26).

Thus, according to Dalidovich's model, the loop length of the normal single knitted structure depends only on yarn diameter.

2.7 Korlinski's loop model

Korlinski (8) suggested a general planar geometrical loop model of the single knitted structure, in which he presumed the elliptical shape of the needle and sinker arc, and jamming of the adjacent loops within the connecting sections of the loop (cf. Figure 14).

The needle loop is an ellipse with the half axes r_1 and r_2. The sinker arc equals the needle arc. The loop legs are connected to the needle and sinker arcs with the straight section ΔB, where the adjacent loops are joined (cf. Figure 14). With a simplified construction, the line of the loop leg is not folded but straight, connecting the needle arc and the sinker arc of the loop.

The simplified loop leg construction, the loop length being (Equation 27) thus (Equation 28), where k_k is the ratio between the half axes of the needle/sinker arc ellipse – $k_k = r_1 / r_2$, the portion of the loop legs joint length within the total loop length is $\varepsilon B = \Delta B / B$, and A is loop width, B loop height, B_1 the vertical distance from the needle to the sinker arc of the same loop, ΔB the vertical portion of the loop leg where the adjacent loops are joined, d_{pr} yarn diameter, r_1 and r_2 the half axes of the needle/sinker arc ellipse, ℓ_1 is the total length of the needle and sinker arcs, and ℓ loop length.

As a special case of the general single knitted loop, Korlinski also developed a theoretical model of the loop from a fully elasticized yarn. He presumed the contact of the needle and sinker arcs of the loops of the subsequent course in a non-stressed state (cf. Figure 15), semicircular shape of the needle and sinker arc, thus $r_2 = r_1$, which follows to $k_k = 1$, and the contact of the adjacent loops in points, thus $\varepsilon B = 0$. In the horizontally extended state, the semicircu-

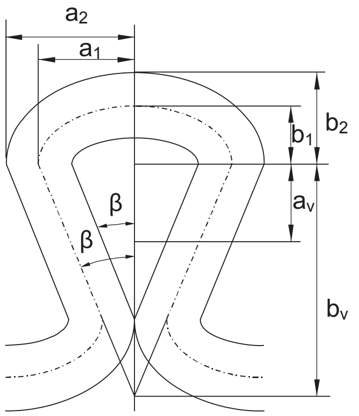

Figure 9: Vekassy's loop model of vertically compact knitted structure (4) – projection onto fabric plane

2.5 Suhov model zanke

Suh [5] se je pri svojih raziskavah ukvarjal s krčenjem bombažnega levo-desnega pletiva zaradi nabrekanja preje pri namakanju in pranju. Vzdolžno krčenje je pojasnjeval s spremembo oblike zanke po namakanju in sušenju, prečno krčenje pa s spremenjenim razmerjem med horizontalno gostoto pletiva in premerom preje.

Tudi Suh je predpostavil, da ima preja v zanki okrogel prerez in konstanten premer. Preje sosednjih zank se dotikajo v točkah M in N. Igelni lok MN je enak platinskemu loku DA z dolžino $L_{i,p}$ in je del kroga s tetivo DA. Razdalja med tetivo MN in vrhom zanke Q je y (sliki 10 in 11). Širina zanke je A. Višina zanke je B. Dolžina kraka zanke je L_k. Krak zanke leži na konkavni ravnini S z radijem ρ. Usločenost zanke je določena z višino zanke B, višino igelnega oz. platinskega loka y in premerom preje d_{pr}. Iz slike 12 je razvidno, da se z zmanjšanjem tetive loka zančne vrste $B + 2y$ in/ali s povečanjem premera preje d_{pr} zmanjša radij loka zančnih krakov ρ in s tem usločenost zanke; s tem se poveča vertikalna gostota pletiva.

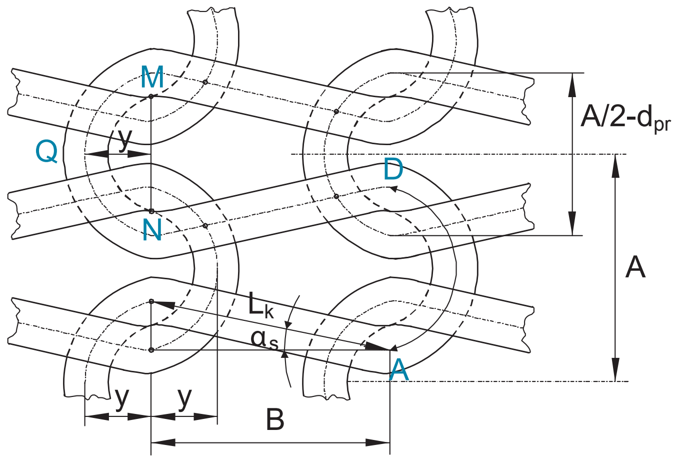

Figure 10: Suh's loop model (5) – projection onto fabric plane

lar shape of the needle and sinker arcs converts into an elliptical shape.

For the model of the loop from a fully elasticized yarn, it holds (cf. Figure 15) (Equation 29).

2.8 Morooka & Matsumoto & Morooka's loop model

Morooka Hi., Matsumoto and Morooka Ha. (9) investigated the shape of the loop in a pantyhose fabric. In their model (cf. Figure 16), they presumed that the knitted fabric tightly covers the rigid cylinder of the infinite length and radius R_{3M}. The total number of wales is n_w, and the total number of courses is n_c. The covered height of the cylinder is h_{3M}.

To simplify the analysis, the two-dimensional shape of the loop was presumed, argued by the fabric thickness being small enough to be ignored in comparison with the length of the needle and sinker loops in the pantyhose fabric. Moreover, the yarn-yarn friction at the loop interlacing points was ignored and the yarn was assumed to be non-extensible.

If the yarn diameter is d_{pr}, the distance between two adjacent wales or the loop width is $A = 2\pi \cdot R_{3M} / n_w$. The distance between the adjacent courses or loop height is $B = h_{3M} / n_c$. From the Figure 16, it can be seen that the model presumes a right angle between the line connecting the centres of the yarn cross-sections at the joint OG_1OF_1 and the line connecting the centres of the yarn cross-sections at the joint OG_1OE_1.

The loop consists of eight sections. ℓ_1, ℓ_3, ℓ_5 and ℓ_7 are circular arcs with the radius d_{pr} and it holds (Equation 30), where ℓ_2, ℓ_4, ℓ_6 and ℓ_8 are straight portions of the loop. The total length of the straight portions of the needle and sinker loop is (Equation 31), while the length of the legs is (Equation 32).

The loop length is (Equation 33), where R_{3M} is the radius of the cylinder covered with the fabric, h_{3M} the height of the cylinder covered with the fabric, n_w the total number of the wales on the cylinder, n_c the total number of the courses on the cylinder, YL the length of the yarn in a course, $\ell = YL / n_w$ is loop length, $A = 2\pi \ R_{3M} / n_w$ is loop width, $B = h_{3M} / n_c$ is loop height and d_{pr} is yarn diameter.

With the maximum value of the cylinder radius R_{maks}, where the fabric tightly covers the cyl-

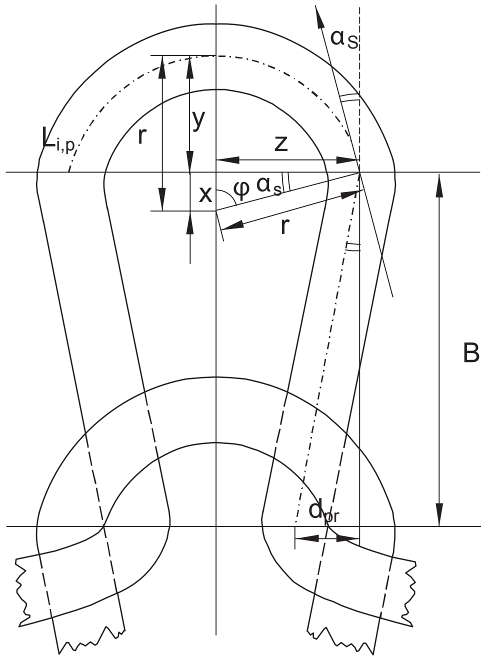

Figure 11: Suh's loop model (5) – needle and sinker arcs, and loop legs

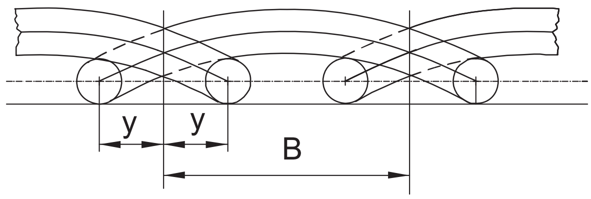

Figure 12: Suh's loop model (5) – loop flexion

Platinski in igelni lok nista polkroga, temveč loka s središčnim kotom $2\varphi < 180°$ (slika 11). Kot med tangento igelnega loka in navpičnico α_S je enak kotu nagnjenosti krakov zanke z dolžino L_k proti navpičnici (sliki 10 in 11). Iz tega sledi, da je višina igelnega oz. platinskega loka $y < r_S$, če je r_S – radij igelnega oz. platinskega loka. Zaradi poenostavitve je Suh pri izračunu dolžine zanke predpostavil, da je dolžina prostorskega igelnega oz. platinskega loka približno enaka projekciji loka v ravnini pletiva, dolžina kraka zanke L_k pa je enaka dolžini loka s polmerom ρ, ker krak zanke L_k leži na konkavni ravnini S s polmerom ρ. Iz slike 11 je razvidno, da je dolžina igelnega oz. platinskega loka $L_{i,p}$:

$$L_{i,p} = 2\, r_S \cdot \varphi \qquad (16)$$

Dolžina kraka zanke L_k je:

$$L_k = 2\,\rho \cdot \sqrt{1 + \left(\frac{d_{pr}}{B}\right)^2} \sin^{-1}\left(\frac{B}{2\,\rho}\right) \qquad (17)$$

inder surface, the loop is maximally extended in the horizontal direction (cf. Figure 17). The sinker loops are in contact with the needle loops of the previous course, thus the loop height is minimal, and $B_{min} = 3d_{pr}$.

The width of the horizontally extended loop A_{maks} is (Equation 34), where $A_{maks} = 2\pi \cdot R_{maks} / n_w$ is the width of the horizontally extended loop, $\ell = YL_{R\,maks} / n_w$ is loop length and d_{pr} is yarn diameter.

With the minimum value of the cylinder radius R_{min}, where the fabric tightly covers the cylinder surface, the loop is maximally extended in the vertical direction (cf. Figure 18). The width of the straight portion of the needle or sinker loop is minimal: $\ell_4 = \ell_8 = d_{pr}$.

The length of the vertically extended loop legs ℓ_6 is maximal. The height of the vertically extended loop is (Equation 35).

The width of the vertically extended loop is (Equation 36), where $A_{min} = 2\pi \cdot R_{min} / n_w$ as the width of the vertically extended loop, $B_{maks} = h_{R\,min} / n_c$ as the height of the vertically extended loop , $\ell = YL_{R\,min} / n_w$ as loop length and d_{pr} is yarn diameter.

3 Discussion on geometrical loop models: open, normal and compact knitted structure

3.1 Peirce's loop model

Apart from the normal knitted structure, Peirce (1) also mathematically described the open knitted structure having the positive values of the inserted yarn segments $\varepsilon \cdot d_{pr}$ and $\xi \cdot d_{pr}$. Under the unchanged basic presumptions, in his model, he also described the compact knitted structure. If the coefficient ξ holds a negative value, also the vertically inserted limb extensions hold a negative value, thus the loop height and the loop leg length are smaller than the ones stated in Equation 2. In order to shorten the loop legs of the normal knitted structure, where the needle and sinker arcs of the adjacent loops are in contact, either the yarn compressibility or the change of the needle and the sinker loop shape is required, which is in contradiction with the basic presumptions of Peirce's model. Peirce (1) only presumed the compact structure in the vertical direction, as he only de-

ter dolžina zanke ℓ:

$$\ell = (A + 2d_{pr}) \cdot \sqrt{1 + \left(\frac{d_{pr}}{B}\right)^2} \cdot \tan^{-1}\left(\frac{B}{2\,\rho}\right) + $$
$$+ 4\,\rho \cdot \sqrt{1 + \left(\frac{d_{pr}}{B}\right)^2} \cdot \sin^{-1}\frac{B}{2\,\rho} \qquad (18),$$

kjer je: ℓ – dolžina zanke, A – širina zanke, B – višina zanke, y – višina igelnega oz. platinskega loka zanke, d_{pr} – debelina preje in ρ – polmer loka zančnih krakov.

2.6 Dalidovičev model zanke

Dalidovičev splošni model zanke [6, 7] je prostorski. Predpostavlja ohlapno strukturo pletiva z razdaljo k med platinskima oz. igelnima lokoma zanke (slika 13).

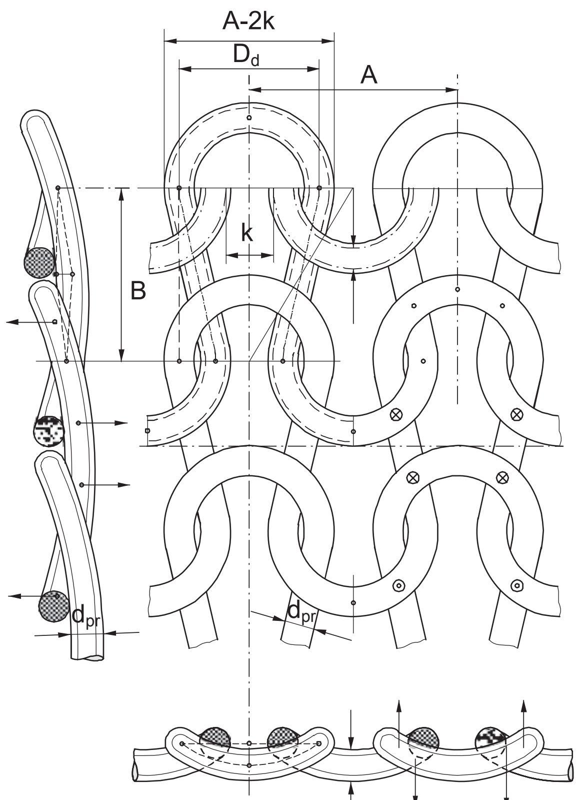

Figure 13: Dalidovich's loop model

Po Dalidoviču je dolžina zanke funkcija širine A in višine zanke B ter premera preje d_{pr}. Poenostavljena dolžina zanke, pri kateri so upoštevani navpični kraki zanke z dolžino, enako višini zanke, je (slika 13):

$$\ell = \pi \cdot D_d + 2\,B \qquad (19).$$

Iz slike 13 je videti, da je:

fined the negative value of the coefficient ξ, but not in the horizontal direction. Since the coefficient ε can only have a positive value, the loop width remains $A = 4d_{pr}$ for the compact structure as well.

Despite the geometrical type of his model, Peirce (1) descriptively presumed the possibility of the stress originating at the loop contact points if there is no yarn sliding occurring at the stress. The loop curves become sharper at contact points, the yarn axes draw nearer, the yarn diameter d_{pr} decreases and the loop length increases. Therefore, Peirce presumed the yarn diameter change, the loop compressibility and length thus increasing during extension. However, he did not describe these changes mathematically.

Peirce (1) did not test the adequacy of his model with laboratory measurements and analyses of the real knitted fabrics. The adequacy of his model was verified through experimental work by Fletcher and Roberts (27, 28, 30). Peirce's loop model was quoted, commented upon or tested by other authors as well, inter alia by Shinn (33), Munden (3), Knapton et al (38) etc.

3.2 Leaf & Glaskin's loop model

Leaf & Glaskin's loop model (2) is composed of four yarn sections, for which the central lines are identical circular arcs. The model is only valid for the radius of the circular arcs $a_{LG} \cdot d_{pr}$ being bigger than 1.5 of the yarn diameter, thus $a_{LG} \geq 1.5$, which means that the possibility of yarn compressibility is excluded from the model. Furthermore, the central angle of the circular arcs 90°< φ < 150° if $a_{LG} \to \infty$. In comparison to Peirce's model and within the frame of quoted limitations, Leaf & Glaskin's model (2) only presumes the normal to open knitted structure, but not the compact knitted structure.

Leaf & Glaskin (2) asserted that the yarn in the loop is not extended at low uniaxial stress until the loop arc join. Leaf & Glaskin (2) also asserted that in most practical cases, the loop length and the yarn diameter do not vary at low uniaxial stresses. The coefficient of the circular arc diameter a_{LG} and the central angle of the circular arc φ modify in such a way that the ratio ℓ / d_{pr} remains a constant. Therefore, Leaf & Glaskin (2) presumed that at a low uniaxial

$$A = 4\,d_{pr} + 2\,k \Rightarrow k = 0.5\,A - 2\,d_{pr} \quad (20),$$

in:

$$D_d = 3\,d_{pr} + k \quad (21).$$

Pri poenostavitvi, da je zanka ploskovna, da sta kraka zanke vzporedna z navpično osjo zanke in imata dolžino B, iz enačb 19–21 sledi:

$$\ell = 1.57\,A + \pi \cdot d_{pr} + 2\,B \quad (22).$$

Če je zanka ploskovna, kraka pa ležita poševno v ravnini pletiva, kot kaže slika 13, velja:

$$\ell = 1.57\,A + \pi \cdot d_{pr} + 2\,B_1 = 1.57\,A + \pi \cdot d_{pr} + 2 \cdot \sqrt{B^2 + d_{pr}^2} \quad (23).$$

Pri prostorski legi krakov zanke, pri čemer je upoštevana poenostavljena lomljena linearna oblika krakov zanke, enačba dobi obliko:

$$\ell = 1.57\,A + \pi \cdot d_{pr} + 2\,B_2 = 1.57\,A + \pi \cdot d_{pr} + 2 \cdot \sqrt{B^2 + 2\,d_{pr}^2} \quad (24),$$

kjer je: ℓ – dolžina zanke, A – širina zanke, B – višina zanke, D_d – polmer središčnega loka igelne/platinske glave, d_{pr} – debelina preje in k – razdalja med igelnimi oz. platinskimi loki zanke.

Če se igelni in platinski loki zank horizontalno in vertikalno stikajo, tj. pri normalni strukturi zanke, je k = 0, zato iz enačbe 20 sledi, da je $A = 4d_{pr}$.

Pri normalni strukturi pletiva tudi velja razmerje:

$$B = \sqrt{(4\,d_{pr})^2 - (2\,d_{pr})^2} = \sqrt{12\,d_{pr}^2} = 2d_{pr} \cdot \sqrt{3} \quad (25),$$

torej je razmerje med višino in širino zanke oz. med horizontalno in vertikalno gostoto pri geometrijskem Dalidovičem modelu zanke koeficient gostote pletiva normalne strukture pletiva C = 0,866. Dolžina zanke normalne strukture pletiva je:

$$\ell = 16.64\,d_{pr} \quad (26).$$

V normalni strukturi levo-desnega pletiva je torej tudi po Dalidovičevem modelu dolžina zanke odvisna le od premera preje.

2.7 Model zanke Korlinskega

Korlinski [8] je predlagal splošen ploskovni geometrijski model zanke levo-desnega pletiva, pri katerem je predpostavil eliptično obliko igelne in platinske glave ter naleganje sosednjih zank v veznih odsekih (slika 14).

Igelna glava je elipsa s polosema r_1 in r_2. Platinska glava je enaka igelni glavi zanke. Krak zanke z igelno in platinsko glavo povezuje ravni odsek ΔB, kjer nalegata sosednji zanki (slika 14). Pri poeno-

stress only the loop extends and not the yarn. In consequence, the yarn diameter does not change, while the shape of the loop and the fabric density do.

In order to compare their model to Peirce's model and to verify the accuracy of their model, Leaf & Glaskin applied the results of the knitted fabrics investigation by Fletcher & Roberts (27, 28, 30). They established that their three-dimensional model is not in accordance with the experimentally defined loop length – in most cases, it overestimates its value. The error increases with the yarn linear density increase, and with the fabric density increase at the constant yarn linear density (2). They forecasted a new loop model with the elliptical projection of the loop curve onto the fabric plane, but they either abandoned the new model development or they never published their results.

3.3 Munden's loop model

Munden's loop model (3) is three-dimensional, the yarn is highly elastic and the friction forces at the loop contact points are small. The shape of the non-stressed loop is defined with the state of minimum energy. The loop adjusts to stress by bending, and after the stress is released, it returns to the state of minimum energy, i.e. to the relaxed state. On the basis of Leaf's elastic theory (37), Munden presumed that the loop shape is exclusively a geometrical property of a knitted structure. Although Munden presented his loop model graphically, he did not define or mathematically present its geometrical shape except for the position of the loop interlacing points and the equality of the needle and sinker loop. He emphasized that the uniformity of the loop geometrical shape is only valid if yarn is not plastically deformed or unrelaxed. On the basis of the theory of similar curves, he derived the equations relating the basic knitted structure parameters of his model: loop length and horizontal, vertical and area density of knitted fabric. This mathematical derivation confirmed the functional dependence which was presumed on the basis of experimental research (Doyle, discussion to the paper) (26).

Munden (3) did not define the numerical values of the constants K_1, K_2, K_3 and K_4. He defined the values of constants experimentally for

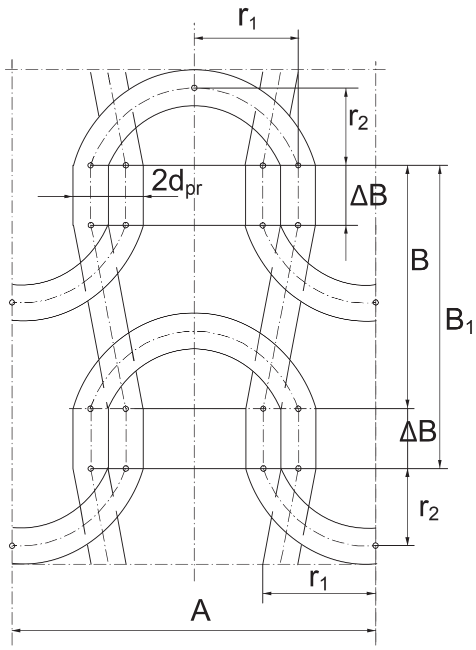

Figure 14: Korlinski's loop model (8)

stavljeni konstrukciji krak zanke ni lomljene oblike, temveč ravno povezuje igelni in platinski lok zanke.

Pri poenostavljeni konstrukciji kraka zanke je dolžina zanke:

$$\ell = \pi\,(r_1 + r_2) + 2\,\sqrt{(B + \Delta B)^2 + d_{pr}^2} \qquad (27),$$

torej:

$$\ell = \frac{\pi}{4} \cdot (A + 2\,d_{pr}) \cdot (1 + k_k) + 2\,\sqrt{B^2 \cdot (1 + \varepsilon B)^2 + d_{pr}^2} \qquad (28),$$

kjer je k_k razmerje polosi elipse igelnega/platinskega loka zanke $k_k = r_1 / r_2$, delež dolžine naleganja sosednjih zank glede na višino zanke pa $\varepsilon B = \Delta B / B$ in A – širina zanke, B – višina zanke, B_1 – navpična razdalja med igelnim in platinskim lokom iste zanke, ΔB – navpični ravni odsek kraka zanke, kjer nalegata sosednji zanki, d_{pr} – premer preje, r_1 in r_2 – polosi elipse igelne/platinske glave zanke, ℓ_1 – skupna dolžina lokov igelne in platinske glave zanke in ℓ – dolžina zanke.

Korlinski je kot poseben primer splošnega modela zanke levo-desnega pletiva razvil tudi teoretični model zanke iz visokoelastičnih prej. Pri tem je v neobremenjenem stanju predpostavil stikanje igelnih lokov in platinskih lokov zank naslednje zančne vrste (slika 15), polkrožno obliko igelnega in platinskega loka zanke, tj. $r_2 = r_1$, iz česar sledi $k_k = 1$ ter točkovno stikanje sosednjih zank, tj. $\varepsilon B = 0$. V horizontalno raztegnjenem stanju polkrožna oblika igelnih in platinskih lokov preide v eliptično.

Za model zanke pletiva iz visokoelastične preje torej velja (slika 15):

yarns of different material composition, and for dry and wet relaxation. The results of the experimental work show that the constants K_1, K_2, K_3 and K_4 exhibit different values for wet and dry relaxed state, respectively, and also that the values for the defined relaxation state are only approximately identical for yarns with different absorbency. The values of the constants K_1, K_2 and K_3 for the wet relaxed knitted fabrics are higher than the values of these constants for the dry relaxed knitted fabrics (3, 39). As the wet relaxed knitted structure is more compact due to the relaxation shrinkage, higher values of Munden constants K_1, K_2 and K_3 at the identical loop length indicate higher fabric density. Therefore, Munden indirectly described the openness and compactness of the knitted structure with his constants, but did not evaluate the types of the structure geometrically and numerically.

3.4 Vekassy's loop model

Vekassy (4) evaluated Dalidovich's (6) idealized planar loop model, where the loop is composed of circular arcs and straight sections joined at points, and the experimental work by Doyle (26), who established that the knitted fabric area density is only dependant on loop length. Vekassy estimated that the advantage of the idealized loop model is its simplicity, but emphasized that it is only a projection of the loop onto the fabric plane which is not a realistic presentation of the loop. He also exposed that the idealized loop model is only valid for the special case of the normal knitted structure, and is thus not generally applicable (4).

In his general loop model, Vekassy (4) described the single structure loop as a real space-curve with minimal mathematical simplifications. Vekassy's general loop model (4) enables the spacing between the needle and sinker arcs in both vertical and horizontal direction; therefore, it assumes the existence of knitted structures from normal to very open with no limits. The equation of the general loop model for the loop length calculation is a very complex mathematical term in which Vekassy's general loop is a line and not a geometrical shape. The radius of the cylinder R on which the loop lies defines the loop flexion, and through the introduction

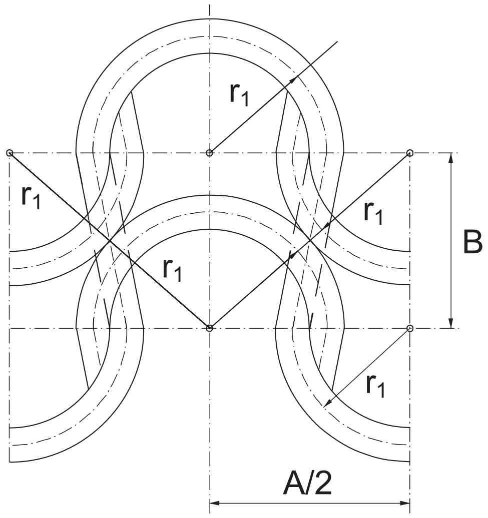

Figure 15: Korlinski's model of loop from fully elasticized yarn (8) – projection of non-stressed loop onto fabric plane

$$B^2 = 2\,A \cdot d_{pr} + (2\,A + d_{pr})^2 \quad (29).$$

2.8 Morookov in Matsumotov in Morookov model zanke

Morooka Hi., Matsumoto in Morooka Ha. [9] so raziskovali obliko zanke v pletivu hlačnih nogavic. V svojem modelu (slika 16) so predpostavili, da je pletivo tesno položeno prek togega valja neskončne dolžine z radijem R_{3M}; skupno število zančnih stolpcev je n_w, skupno število zančnih vrst je n_c. Višina valja, prek katere sega pletivo, je h_{3M}.

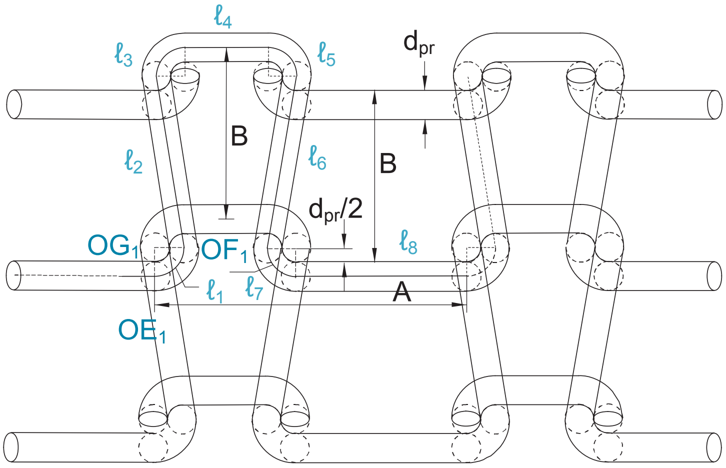

Figure 16: Morooka & Matsumoto & Morooka's general loop model

Za poenostavitev analize so predpostavili dvodimenzionalno obliko zanke, ki so jo utemeljili z zanemarljivo debelino pletiva v primerjavi z dolžino igelne in platinske zanke pri pletivu za hlačne nogavice. Zanemarili so tudi trenje preja-preja na stičnih točkah zanke ter predpostavili, da je preja neraztezna.

of the known parameter – yarn diameter, it also defines the fabric thickness. Therefore, Vekassy's general loop model is valid for **normal** *to* **open** *structure. However, for the structure type definition, also the yarn diameter d_{pr} and the parameters determining the loop line length need to be introduced.*

Vekassy (4) introduced the second parameter of the realistic knitted fabric, thus yarn diameter d_{pr} for two special structural cases, i.e. for normal and for compact structure of the single knitted fabric. Dalidovich (6) considered the loop projection onto the fabric plane in his loop length calculation, and he established the ratio $\ell / d_{pr} = 16.64$. Vekassy's ratio of the loop length of the normal knitted structure and yarn diameter is bigger than Dalidovich's, taking into consideration the loop space-curve, and attains that $\ell / d_{pr} = 17.33$. The difference is 4.14% (4).

Vekassy (4) also presumed the compact structure as a special case of his general loop model. The normal and the compact knitted structure have equal loop width A when the yarn diameter is equal, thus also the density of the knitted fabric is equal. The loop height B of the normal and compact knitted structure differs due to the different shape of the needle and sinker arc, which are semicircular or elliptic, respectively. Consequently, with equal loop height and yarn diameter, also the loop length and the fabric density coefficient differ. The linear loop module of the compact structure $\ell / d_{pr} = 13.40$ (4), which means that with an identical yarn diameter, the difference between the loop length of the normal knitted structure and the compact knitted structure is 22.7% (4).

3.5 Suh's loop model

Suh (5) studied exclusively cotton knitted fabrics and structural changes of the loop geometry due to yarn swelling in a wet state. He presumed that the yarn cross-section is circular and that the yarn diameter is a constant. In his model (5), he anticipated that the central angle of the needle and sinker arc is $2\varphi \leq \pi$, that the structure is normal, thus the needle and sinker arcs of the adjacent courses are in contact, or the structure is open, thus the needle and sinker arcs of the adjacent courses are not in contact. This signifies that it is possible to plan normal

Če je premer preje d_{pr}, je razdalja med sosednjima zančnima stolpcema oz. širina zanke $A = 2\pi \cdot R_{3M} / n_w$. Razdalja med zančnimi vrstami oz. višina zanke je $B = h_{3M} / n_c$. Na sliki 16 je videti, da model predpostavlja pravi kot med premico, ki povezuje središči prerezov preje na stičišču prej OG_1OF_1 in premico, ki povezuje središči prerezov preje na stičišču prej OG_1OE_1.

Zanko sestavlja osem odsekov. ℓ_1, ℓ_3, ℓ_5 in ℓ_7 so krožni loki s polmerom d_{pr}, pri čemer velja:

$$\ell_1 = \ell_3 = \ell_5 = \ell_7 = \frac{\pi \cdot d_{pr}}{2} \quad (30),$$

ℓ_2, ℓ_4, ℓ_6 in ℓ_8 so ravni deli zanke, pri čemer velja, da je vsota dolžine ravnih delov igelne in platinske zanke:

$$\ell_4 + \ell_8 = \left(\frac{2\pi \cdot R_{3M}}{n_w}\right) - 2\, d_{pr} = A - 2\, d_{pr} \quad (31),$$

dolžina krakov zanke pa:

$$\ell_2 = \ell_6 = \sqrt{\left(d_{pr}^2 + \left(\frac{h_{3M}}{n_c}\right)^2\right)} = \sqrt{d_{pr}^2 + B^2} \quad (32).$$

Dolžina zanke je:

$$\ell = \frac{YL}{n_w} = \frac{2\,\pi \cdot R_{3M}}{n_w} + 2 \cdot \left(\sqrt{d_{pr}^2 + \left(\frac{h_{3M}}{n_c}\right)^2} + 2.14\, d_{pr}\right) = \quad (33),$$

$$= A + 2 \cdot \left(\sqrt{d_{pr}^2 + B^2} + 2.14\, d_{pr}\right)$$

kjer je: R_{3M} – polmer valja, pokritega s pletivom, h_{3M} – višina valja, pokritega s pletivom, n_w – skupno število zančnih stolpcev na valju, n_c – skupno število zančnih vrst na valju, YL – dolžina niti v zančni vrsti, $\ell = YL / n_w$ – dolžina zanke, $A = 2\pi \cdot R_{3M} / n_w$ – širina zanke, $B = h_{3M} / n_c$ – višina zanke in d_{pr} – debelina preje.

Pri maksimalni vrednosti premera valja R_{maks}, pri kateri se pletivo tesno prilega površini valja, je zanka maksimalno prečno razteg-

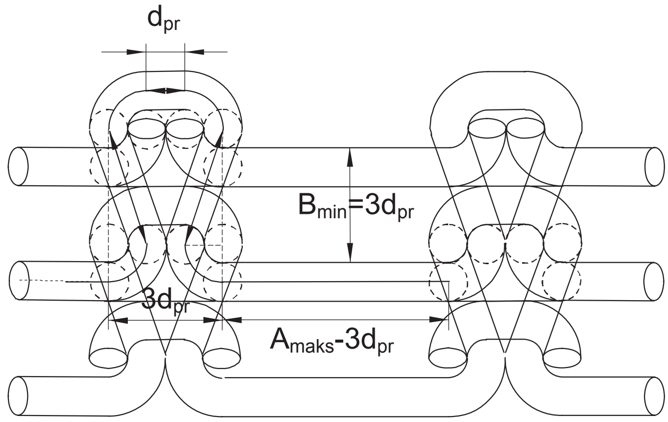

Figure 17: Morooka & Matsumoto & Morooka's model of horizontally extended loop

to infinitely open knitted structures by varying the central angle of the needle and sinker arc, acquiring different ratios between the vertical and horizontal knitted fabric density. That corresponds to the structure of a real knitted fabric.

In his model, Suh (5) also presumed that the angle of the inclination of the loop legs α_S is a complementary angle to the angle φ, which is a half of the central angle of the needle and sinker arcs. It signifies that with the central angle of the needle and sinker arc decrease, the loop leg length decreases and the angle of the crossing from the loop arc to the loop leg increases. The fabric vertical density increases. When the central angle of the needle and sinker arc is $2\varphi = \pi / 2$, the angle between the tangent of the needle and the sinker arc and the loop leg is a right angle, which is an extensive simplification of the real loop shape. If the central angle is $2\varphi = \pi$, the needle and sinker arc become semicircles, and the knitted structure becomes infinitely open, as the loop legs become vertical and infinitely long. Suh did not limit the values of the central angle of the needle and sinker arcs as Leaf & Glaskin (2) did.

Suh (5) presumed that the yarn length changes due to the wet relaxation are negligible. He presumed that the change of the loop structure is only influenced by the yarn swelling in transversal direction, thus only the yarn diameter changes with swelling. With his estimation that the loop width is equal or less than quadruple yarn diameter, he descriptively presumed the yarn compressibility.

3.6 Dalidovich's loop model

Dalidovich's general loop model (7) is three-dimensional; however, in his mathematical derivation of the loop length ℓ, Dalidovich simplified the dimensional parameters of the loop composing elements. The length of the needle and sinker arc was calculated from the projection of the loop onto the fabric plane, while for the loop leg calculation different simplifications concerning leg inclination were taken into consideration. For the general Dalidovich's model (7), the loop length is defined with three parameters, i.e. loop width A, loop height B and yarn diameter d_{pr}. It signifies that the loop length of

njena (slika 17). Platinske zanke se dotikajo igelnih zank prejšnje zančne vrste, zato je vrednost višine zanke minimalna, in je $B_{min} = 3d_{pr}$.

Širina prečno raztegnjene zanke A_{maks} je:

$$A_{max} = \frac{2\,\pi \cdot R_{max}}{n_w} = \frac{Y \cdot L_{R\,max}}{n_w} - 10.6\; d_{pr} \qquad (34),$$

kjer je $A_{maks} = 2\pi \cdot R_{maks} / n_w$ – širina prečno raztegnjene zanke, $\ell = YL_{R\,maks} / n_w$ – dolžina zanke in d_{pr} – premer preje.

Pri minimalni vrednosti premera valja R_{min}, pri kateri se pletivo tesno prilega valju, je zanka maksimalno vzdolžno raztegnjena (slika 18). Širina ravnega dela igelne oz. platinske zanke je minimalna: $\ell_4 = \ell_8 = d_{pr}$

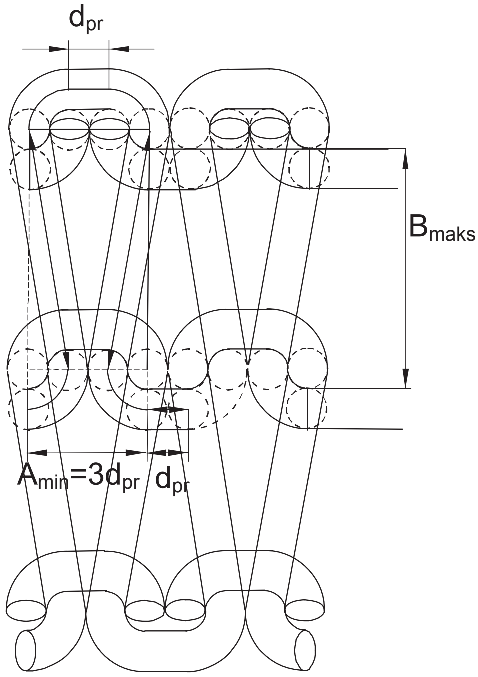

Figure 18: Morooka & Matsumoto & Morooka's model of vertically extended loop

Dolžina krakov vzdolžno raztegnjene zanke ℓ_6 je maksimalna. Višina vzdolžno raztegnjene zanke je:

$$B_{max} = \frac{h_{Rmin}}{n_c} = \sqrt{\left(\frac{YL_{R\,min}}{2n_w} - 4.14\; d_{pr}\right)^2 - d_{pr}^2} \qquad (35).$$

Širina vzdolžno raztegnjene zanke je:

$$A_{min} = \frac{2\,\pi \cdot R_{min}}{n_w} = 4\; d_{pr} \qquad (36),$$

kjer je: $A_{min} = 2\pi \cdot R_{min} / n_w$ – širina vzdolžno raztegnjene zanke, $B_{maks} = h_{R\,min} / n_c$ – višina vzdolžno raztegnjene zanke, $\ell = YL_{R\,min} / n_w$ – dolžina zanke in d_{pr} – premer preje.

the fabrics knitted from yarns of various thicknesses but of equal vertical and horizontal density varies.

The projection of Dalidovich's loop model (7) onto the fabric plane describes an open structure, defined with the horizontal distance k between the needle and sinker arcs, and the loop height B. From the side and bottom view of the general Dalidovich's model (7), it can be seen that in the interlacing points, the yarns are in contact and that the fabric thickness is twice the yarn thickness. From the side view, the knitted structure is not open but normal.

Dalidovich (7) described the normal structure as a special case of the general, open structure. The loop length of the normal structure depends only on yarn diameter d_{pr}. The difference between Dalidovich's value $\ell = 16.64d_{pr}$ (6) and Perice's value $\ell = 16.66d_{pr}$ (1) is only minimal, although Dalidovich's central angle of the needle and sinker arc is π, while Peirce's central angle is bigger than π. With Peirce's model, the loop legs lie tangential to the needle and sinker arcs, which is not the case with Dalidovich's model. Therefore, the length of the needle and sinker arc is bigger and the loop leg length is smaller at Peirce's model comparing to Dalidovich's model. For the normal structure of both models, the loop width $A = 4d_{pr}$ and the loop height $B = 2d_{pr} \cdot 3^{1/2}$. This signifies that the knitted fabric density coefficient is equal for the normal structure of both models.

3.7 Korlinski's loop model

Korlinski's loop model (8) is planar, the loop leg being shown as a folded line due to the jamming of the needle and sinker arcs of the adjacent courses. In the loop length calculations, the simplified, thus straight, line of the loop leg is considered. The loop model is general and anticipated for all kind of materials and knitted fabric states (8).

In Korlinski's general model, the needle and sinker arc have an elliptical shape. The semicircular shape of the needle/sinker arc is only valid for the special case of the fully elasticized yarn (8). In this case, Korlinski's loop model (8) resembles Suh's loop model (2), discussing the semicircular shape of the needle/sinker arc only as a special, extreme case. The difference

3 Razprava o geometrijskih modelih zanke: ohlapna, normalna in zbita struktura pletiva

3.1 Peirceov model zanke

Poleg *normalne* je Peirce [1] matematično opisal tudi *ohlapno stukturo pletiva* s pozitivnimi vrednostmi podaljškov preje $\varepsilon \cdot d_{pr}$ in $\xi \cdot d_{pr}$. Ob nespremenjenih navedenih temeljnih predpostavkah je v svojem modelu opisal tudi *zbito strukturo pletiva*. Če ima koeficient ξ negativno vrednost, imajo tudi navpični podaljški krakov zanke negativne vrednosti in sta višina zanke ter dolžina krakov zanke manjši od tistih v enačbi 2. Za skrajšanje krakov zanke normalne strukture pletiva, pri kateri se igelni in platinski loki sosednjih zančnih vrst stikajo, je potrebna bodisi stisljivost preje bodisi sprememba oblike igelne in platinske zanke, kar je v nasprotju s temeljnimi predpostavkami Peirceovega modela. Zbitost je Peirce [1] predvidel le v vertikalni smeri, saj je definiral le negativno vrednost koeficienta ξ, ne pa tudi v horizontalni smeri: koeficient ε ima lahko le pozitivno vrednost in širina zanke zato tudi pri zbitem pletivu ostaja $A = 4d_{pr}$.

Peirce [1] je kljub geometrijski naravi svojega modela opisno predvidel možnost nastanka napetosti v stičnih točkah zanke, če pri obremenjevanju ne pride do drsenja niti; krivulji zanke v stičnih točkah postaneta ostrejši, osi niti se približata, premer preje d_{pr} se zmanjša, dolžina zanke pa poveča. Peirce je torej predvidel spremembo premera preje, tj. stisljivost in povečanje dolžine zanke pri raztezanju, vendar teh sprememb ni matematično opisal.

Peirce [1] ustreznosti svojega modela ni preskusil z laboratorijskimi analizami dejanskih pletiv. Ustreznost modela je potrdilo eksperimentalno delo Fletcherja in Robertsa [27, 28, 30]. Peirceov model zanke [1] so navajali, komentirali oz. preskušali tudi drugi avtorji: Shinn [33], Munden [3], Knapton et al. [38] in drugi.

3.2 Leaf-Glaskinov model zanke

Model zanke Leafa in Glaskina [2] sestavljajo štirje odseki preje, pri katerih so središčnice enaki krožni loki. Model velja le pri polmeru krožnih lokov $a_{LG} \cdot d_{pr}$, ki je večji od 1,5 premera preje, tj. $a_{LG} \geq 1{,}5$, kar pomeni, da model izključuje možnost stisljivosti preje. Velja tudi, da je središčni kot krožnih lokov $90° < \varphi < 150°$, če je $a_{LG} \rightarrow \infty$. V primerjavi s Pierceovim modelom torej Leaf-Glaskinov model [20] v okvirih navedenih omejitev opisno predvideva *normalno* in *ohlapno strukturo pletiva*, ne pa tudi zbite strukture.

Leaf in Glaskin [2] sta trdila, da se preja v zanki pri majhnih enoosnih obremenitvah ne razteza, dokler se loki zanke ne stikajo. Leaf in Glaskin [2] sta tudi trdila, da se v večini praktičnih primerov pri majhnih enoosnih obremenitvah ne spremenita dolžina zanke in premer preje; koeficient polmera krožnega loka a_{LG} in središčni kot krožnega loka φ se spremenita tako, da ostane razmerje ℓ / d_{pr} konstantno. Leaf in Glaskin [2] sta torej predpostavila, da se pri *between the two models is that with Suh's loop model (2), the needle and sinker arc are parts of a circle, while with Korlinski's model (8), the needle and sinker arcs are elliptical. Beside Suh (2), also Vekassy (4) proposed a non-cylindrical shape of the needle and sinker arc before Korlinski. Korlinski's model (8) differs from Vekassy's model (4), as Vekassy (4) presents the circular shape of the needle and sinker arc in the projection onto the fabric plane for his general loop model, and the elliptical shape only in the case of the compact structure. Korlinski, on the contrary, defines in his general model the elliptical shape, while the circular shape is only valid as a special case of fully extended yarn. Vekassy's (4) and Korlinski's (8) model also differ in that Vekassy's model is three-dimensional, while Korlinski's model is two-dimensional. Korlinski (8) was the only author among the presented authors of geometrical models who presumed the linear joint of the needle and sinker arcs of the adjacent courses; the joint was defined as ΔB, while the portion of the loop legs which are joined in the relation to loop height was defined as εB = ΔB / B. Korlinski assumed yarn compressibility neither in his general nor in his special case of the fully elasticized yarn.*

In his general loop model, Korlinski (8) defined the open knitted structure, as the needle and sinker arcs are in contact neither in the vertical nor in the horizontal direction. The structure with the joint needle and sinker arc of the adjacent courses, i.e. the vertically normal structure, was foreseen only in the case of the fabric made from fully elasticized yarns. Korlinski referred to the compact structure when quoting Nawrocki's loop model (40), however, he did not develop his own loop model of the compact structure.

3.8 Morooka & Matsumoto & Morooka's loop model

Contrary to other authors of geometrical loop models, Morooka Hi., Matsumoto & Morooka Ha. (9) oriented in their study exclusively to the analysis of the single circular hosiery fabric in which the loop in the extended, strained state captures the characteristic extended shape. According to the authors' statements, the needle arc with the loop legs represents only a minor

majhnem enoosnem obremenjevanju razteza le zanka, ne pa tudi preja, ter da se pri tem premer preje ne spremeni, spremenita pa se oblika zanke in gostota pletiva.

Za primerjavo s Peirceovim modelom ter potrditev pravilnosti svojega modela sta Leaf in Glaskin [2] uporabila rezultate raziskav pletiv Fletcherja in Robertsa [27, 28, 30]. Ugotovila sta, da se njun tridimenzionalni model ne sklada z eksperimentalno določeno dolžino zanke; večinoma jo precenjuje, napaka pa narašča z naraščanjem dolžinske mase preje ter z naraščanjem gostote pletiva pri konstantni dolžinski masi preje [2]. Napovedala sta nov model zanke z eliptično projekcijo zančne krivulje v ravnini pletiva, vendar sta razvoj novega modela opustila oz. svojih izsledkov nista objavila.

3.3 Mundenov model zanke

Mundenov model zanke [3] je prostorski, preja je visokoelastična, torne sile na stičnih točkah so majhne. Obliko neobremenjene zanke določa stanje minimalne energije; obremenitvam se zanka prilagaja z upogibanjem, po prenehanju pa se vrača v stanje minimalne energije, tj. relaksirano stanje. Munden je na podlagi Leafove teorije elastike predpostavil, da je oblika zanke izključno geometrijska lastnost zančne strukture [37]. Čeprav je Munden grafično prikazal svoj model zanke, njene geometrijske oblike, razen položaja stičnih točk zanke ter enakosti igelne in platinske zanke, je ni definiral, niti je ni matematično opisal. Poudaril je, da enovitost geometrijske oblike zanke velja le, če preja ni trajno plastično deformirana oz. nerelaksirana. Na podlagi geometrije podobnih krivulj je izpeljal enačbe, ki povezujejo temeljne parametre pletiva njegovega modela: dolžino zanke ter horizontalno, vertikalno in ploskovno gostoto pletiva. Ta matematična izpeljava je potrdila funkcijsko odvisnost, predpostavljeno na podlagi eksperimentalnih raziskav (Doyle, razprava k članku) [26].

Munden [3] ni podal numeričnih vrednosti konstant K_1, K_2, K_3 in K_4; vrednosti konstant za mokro in suho relaksacijo za preje različne surovinske sestave je določil eksperimentalno. Rezultati eksperimentalnega dela kažejo, da so konstante K_1, K_2, K_3 in K_4 različne za mokro in suho relaksirano stanje ter da so pri določenem relaksacijskem stanju le približno enake za različno hidrofilne preje. Vrednosti konstant K_1, K_2 in K_3 za mokro relaksirana pletiva so višje od vrednosti teh konstant za suho relaksirana pletiva [3, 39]. Ker je mokro relaksirana struktura pletiva zaradi relaksacijskega krčenja bolj zbita, višje vrednosti Mundenovih konstant K_1, K_2 in K_3 pri enaki dolžini zanke pomenijo večjo gostoto pletiva. Munden je torej s svojimi konstantami posredno opisal zbitost in ohlapnost strukture, ni pa vrste strukture geometrijsko in numerično ovrednotil.

3.4 Vekassyjev model zanke

Vekassy [4] je ocenil idealizirani ploskovni model zanke Dalidoviča [6], pri katerem je zanka sestavljena iz krožnih lokov in ravnih delov, ki se med seboj točkovno stikajo, in eksperimentalno delo *portion of the total loop length, while the sinker arc represents the major portion. Although Morooka & Matsumoto & Morooka's loop model (9) represents the loop as an element of the knitted fabric lying on the cylinder that is a part of the three-dimensional structure, in reality, it is a planar loop model. The authors base this statement with the negligible fabric thickness in relation to the needle and sinker arc within the hosiery fabric. The assumed inextensibility of the yarn (9) signifies that the loop shape, thus the loop width and the loop height change during the imposing stress to the knitted fabric; simultaneously, the loop length remains unchanged, as the loop model is planar.*

Morooka & Matsumoto & Morooka (9) presumed an open structure for the general loop shape in which the needle and sinker arcs are in contact neither in the vertical nor in the horizontal direction. With the rename of the loop parameters: loop length $\ell = YL / N_w$, loop width $A = 2\pi \cdot r / n_w$ and loop height $B = (d^2 + (h / n_c)^2)^{1/2}$, the general equation for the loop length calculation (i.e. Equation 33) is (Eqaution 37). The equation shows the loop length dependence on the loop width, loop height and yarn diameter similar to Peirce's loop model of the open structure (cf. Equation 6), and Dalidovich's general loop model (cf. Equation 22). The coefficients of the presented three models differ, since Dalidovich (7) assumes the semicircular shape of the needle and sinker arc, Peirce (1) assumes the straight yarn portions in the semicircular loop crown and straight loop legs, and Morooka & Matsumoto & Morooka (9) assume straight yarn portions only within the needle and sinker arc of the loop.

Morooka & Matsumoto & Morooka (9) also mathematically defined the loop parameters at extreme conditions, thus at maximal horizontal (transversal) and maximal vertical (longitudinal) fabric extension. Since according to authors' presumptions the yarn is not extensible, the yarn diameter and the loop length do not change during extension. The maximal horizontal extension leads to the vertical normal structure, thus to the joined needle and sinker arcs of the adjacent courses, while the structure remains open in the vertical direction, thus the adjacent wales do not join. The maximal ver-

Doyla [26], ki je ugotovil, da je ploskovna gostota pletiva odvisna izključno od dolžine zanke. Vekassy je ocenil, da je prednost idealiziranega modela zanke njena enostavnost, vendar je poudaril, da gre za projekcijo zanke v ravnini, ki zanke ne opisuje realno. Poudaril je tudi, da idealizirani model zanke velja le za poseben primer normalnega pletiva, torej ni splošno uporaben [4].

Vekassy [4] je v svojem splošnem modelu z minimalnimi matematičnimi poenostavitvami opisal zanko levo-desnega pletiva kot realno prostorsko krivuljo. Vekassyjev splošni model zanke [4] omogoča razmik med igelnimi in platinskimi loki zanke v prečni in vzdolžni smeri, torej predvideva strukturo pletiva od normalne do ohlapne brez omejitev. Enačba splošnega modela za izračun dolžine zanke je zapleten matematični izraz, pri čemer je Vekassyjeva splošna zanka črta in ne geometrijsko telo. Polmer kroga R, na katerem leži zanka, definira usločenost zanke in ob vpeljavi znanega premera preje podaja debelino pletiva. Vekassyjev splošni model zanke torej velja za *normalno* do *ohlapno strukturo pletiva*, vendar je treba za opredelitev vrste strukture poleg parametrov, ki določajo dolžino črtne zanke, definirati tudi premer preje d_{pr}.

Vekassy [4] je drugi parameter realnega pletiva, tj. premer preje d_{pr}, vpeljal v dveh posebnih primerih strukture: pri normalni in zbiti strukturi levo-desnega pletiva. Dalidovič [6] je pri izračunu dolžine zanke upošteval projekcijo zanke v ravnini pletiva in podal razmerje $\ell / d_{pr} = 16{,}64$. Vekassyjevo razmerje dolžine zanke normalnega pletiva in premera preje je ob upoštevanju prostorske krivulje zanke večje od Dalidovičevega [6] in je: $\ell / d_{pr} = 17{,}33$. Razlika je 4,14 % [Vekassy 4].

Vekassy [4] je kot poseben primer svojega splošnega modela zanke predvidel tudi *zbito strukturo pletiva*. Normalna in zbita struktura pletiva imata pri enakem premeru preje enako širino zanke A, tj. tudi horizontalno gostoto pletiva. Višini zanke normalnega in zbitega pletiva se zaradi različne oblike igelne/platinske glave, polkrožne oz. eliptične, razlikujeta, s tem pa se pri enakem premeru preje in širini zanke razlikujeta tudi dolžina zanke in koeficient gostote pletiva. Dolžinski modul zanke zbitega pletiva je $\ell / d_{pr} = 13{,}40$ [4], kar pomeni, da je pri enakem premeru preje razlika med dolžino zanke normalnega in zbitega pletiva Vekassyjevega modela zanke [4] 22,7 %.

3.5 Suhov model zanke

Suh [5] se je ukvarjal izključno z bombažnim pletivom in strukturnimi spremembami geometrije zanke zaradi nabrekanja preje v mokrem stanju. Pri tem je predpostavil, da ima preja okrogel prerez in konstanten premer. V svojem modelu [5] je predvidel, da imata igelni in platinski lok središčni kot $2\varphi \leq \pi$ in da je struktura pletiva normalna, pri čemer se igelni in platinski loki sosednjih zančnih vrst stikajo, ali ohlapna, pri čemer se igelni in platinski loki sosednjih zančnih vrst ne stikajo. To pomeni, da je s spremembo središčnega kota igelnega in platinskega loka mogoče projektirati od *normalne* do neskončno *ohlapne strukture pletiva* z *tical extension leads to the horizontal normal structure, thus to joined adjacent wales, while the knitted structure remains open in the vertical direction, thus the needle and sinker arcs of the adjacent courses do not join.*

4 Conclusions

The geometrical loop models by Peirce (1), Leaf & Glaskin (2), Munden (3), Vekassy (4), Suh (5), Dalidovich (6, 7), Morooka & Matsumoto and Morooka (9) are based on the presumptions that yarn diameter is a constant, the yarn cross-section is circular, that the yarn is non-extensible and completely flexible. The real and especially contemporary knitted fabrics are complex material which does not meet the idealized presumptions.

Moreover, the geometrical models explicitly or implicitly presume that the geometrical shape of the loop is constant for each model. Nevertheless, under special conditions, some of the models presume:

- *yarn diameter change – Peirce (1) and Dalidovich (7) when the knitted fabric is subjected to stress,*
- *loop shape change – Peirce (1) and Vekassy (4) when the open or compact structure type changes, Suh (5) at wetting and washing, Dalidovich (7), Korlinski (8), and Morooka & Matsumoto & Morooka (9) when the knitted fabric is subjected to stress,*
- *yarn extensibility – Dalidovich (7).*

In most cases, within the frame of basic assumptions, the geometrical loop models are general, i.e. Leaf & Glaskin's (2), Munden's (3), Vekassy's (4), Suh's (5), Dalidovich's (7), Korlinski's (8), Morooka & Matsumoto & Morooka's (9) loop model. Or they define the structure with the joined needle and sinker arcs of the adjacent loops, thus the normal knitted structure, i.e. Peirce's loop model (1). Some of them assume the individual structure type as a special case, e.g. Peirce (1) describes the open structure, Vekassy (4) and Dalidovich (6) describe the normal structure, and Peirce (1) and Vekassy (4) describe the compact structure.

Due to their simplicity, Peirce's, Dalidovich's and Munden's loop model are the most usable for weft knitted structure planning and analy-

različnim razmerjem med vertikalno in horizontalno gostoto pletiva, kar ustreza strukturi realnega pletiva.

Suh [5] je v svojem modelu tudi predpostavil, da je kot nagnjenosti krakov zanke α_S komplementarni kot φ, tj. polovice središčnega kota igelnega/platinskega loka, kar pomeni, da se z manjšanjem središčnega kota igelnega/platinskega loka krajša dolžina krakov zanke in povečuje kot prehoda igelnega oz. platinskega loka v kraka; vertikalna gostota pletiva se čedalje bolj povečuje. Pri središčnem kotu igelnega/platinskega loka $2\varphi = \pi / 2$ je med tangento igelnega oz. platinskega loka in krakom zanke pravi kot, kar je zelo groba poenostavitev dejanske oblike zanke. Če je velikost središčnega kota $2\varphi = \pi$, sta igelni in platinski lok polkroga, struktura pletiva pa neskončno ohlapna, saj sta kraka zanke navpična in neskončno dolga. Suh ni omejil vrednosti središčnega kota igelnega oz. platinskega loka 2φ, kot sta to npr. storila Leaf in Glaskin [2].

Suh [5] je predpostavil, da so dolžinske spremembe preje zaradi mokre relaksacije zanemarljive. Predpostavil je, da na spremembo strukture zanke vpliva le nabrekanje preje v prečni smeri, tj. da se pri nabrekanju spremeni le premer preje. Z oceno, da je širina zanke enaka ali manjša od štirikratnega premera preje po nabrekanju, je opisno predvidel stisljivost preje.

3.6 Dalidovičev model zanke

Dalidovičev splošni model zanke zanke [6] je prostorski, vendar je Dalidovič v svoji matematični izpeljavi dolžine zanke ℓ poenostavil dimenzijske parametre sestavnih delov zanke. Dolžino igelnega oz. platinskega loka je izračunal iz projekcije zanke v ravnini pletiva, za izračun dolžine kraka zanke pa je upošteval različne poenostavitve glede nagnjenosti krakov zanke. Dolžina zanke je pri splošnem Dalidovičevem modelu zanke [6] določena s tremi parametri: širino zanke A, višino zanke B in debelino preje d_{pr}, kar pomeni, da ima pletivo, spleteno iz različno debele preje, a z enako vertikalno in horizontalno gostoto, različno dolžino zanke.

Projekcija zanke Dalidovičevega modela [6] v ravnini pletiva opisuje *ohlapno strukturo*, ki je določena s horizontalno razdaljo k med igelnimi oz. platinskimi loki in višino zanke B. Iz narisa in stranskega risa splošnega Dalidovičevega modela zanke [6] je videti, da se v veznih točkah zanke niti dotikajo, torej je debelina pletiva enaka dvakratni debelini preje, struktura pa v vzdolžnem prerezu po debelini ni ohlapna, temveč normalna.

Dalidovič [6] je *normalno strukturo* pletiva opisal kot poseben primer splošne, ohlapne strukture. Dolžina zanke normalne strukture je odvisna le od debeline preje d_{pr}. Dalidovičeva vrednost $\ell = 16{,}64 d_{pr}$ [6] se od Peirceove vrednosti $\ell = 16{,}66 d_{pr}$ [1] le minimalno razlikuje, čeprav je pri Dalidovičevem modelu središčni kot igelnega/platinskega loka π, pri Peirceovem pa večji od π; pri Peirceovem modelu kraka zanke ležita tangencialno na igelni/platinski lok, pri Dalidovičevem modelu pa ne. Zato je dolžina igelnega/platinskega loka pri Peirceovem modelu zanke večja, dolžina kraka zanke pa *sis. They define normal, open and under limited conditions also compact structure. They can be used mostly for the study of the knitted fabrics made from conventional yarns. The compact structure, which is nowadays characteristic of knitted fabrics with incorporated elasticized core, can be represented with Munden's constants which have significantly higher values compared to those representing the normal to open structure. Vekassy's model presents the real, three-dimensional knitted structure most adequately; nevertheless, it is generally too complex for fabric planning.*

manjša kot pri Dalidovičevem modelu. Pri obeh modelih za normalno strukturo pletiva velja, da je širina zanke $A = 4d_{pr}$ ter da je višina zanke $B = 2d_{pr} \cdot 3^{1/2}$, kar pomeni, da je koeficient gostote pletiva za normalno strukturo obeh modelov enak.

3.7 Model zanke Korlinskega

Model zanke Korlinskega [8] je ploskoven, pri čemer je krak zanke zaradi naleganja igelnih in platinskih glav sosednjih zančnih vrst prikazan kot lomljena črta. V izračunih dolžine zanke je upoštevana poenostavljena, tj. ravna nelomljena oblika kraka zanke. Model zanke je splošen in je predviden za vse materiale in stanja pletiva [8].

Igelna in platinska glava zanke imata v splošnem modelu Korlinskega eliptično obliko; polkrožna oblika igelne/platinske glave velja le za poseben primer pletiva iz zelo raztezne preje [8]. Model zanke Korlinskega je v tem podoben Suhovemu modelu zanke [2], ki obravnava polkrožno obliko igelne/platinske glave le kot poseben, mejni primer; razlikujeta se v tem, da je pri Suhovem modelu zanke [2] igelni/platinski lok del krožnice, pri Korlinskem [8] pa gre za eliptično obliko igelne/platinske glave. Poleg Suha [2] je nekrožno obliko igelne/platinske glave med obravnavanimi avtorji pred Korlinskim [8] predvidel tudi Vekassy [4]. Model Korlinskega [8] se od Vekassyjevega [4] razlikuje po tem, da Vekassy podaja krožno obliko igelne/platinske glave v projekciji v ravnini pletiva za svoj splošni model zanke, eliptično pa le pri zbitem pletivu, medtem ko Korlinski nasprotno v splošnem modelu podaja eliptično obliko, krožna pa velja le kot poseben primer pletiva iz zelo raztezne preje. Modela zanke Vekassyja [4] in Korlinskega [8] se razlikujeta tudi po tem, da je Vekassyjev prostorski, model Korlinskega pa ploskovni.

Korlinski [8] je edini med obravnavanimi avtorji geometrijskih modelov zanke predvidel črtno naleganje igelnih in platinskih glav zank sosednjih zančnih vrst; opredelil ga je kot ΔB, delež krakov zank, ki nalegata glede na višino zanke, pa kot $\varepsilon B = \Delta B / B$. Korlinski niti v svojem splošnem niti v posebnem primeru modela zanke za pletivo iz zelo raztezne preje [8] ni predvidel stisljivosti preje.

Korlinski [8] je v svojem splošnem modelu opredelil *ohlapno strukturo* pletiva, saj se igelne in platinske glave modela zanke ne stikajo niti v vertikalni niti v horizontalni smeri. Strukturo, pri kateri se stikajo igelni in platinski loki sosednjih zančnih vrst, torej *vertikalno normalno strukturo*, je predvidel le pri pletivu iz zelo elastičnih prej. Zbito strukturo je Korlinski omenil, ko je citiral model Nawrockega [40], lastnega modela zanke zbite strukture pa ni razvil.

3.8 Morookov in Matsumotov in Morookov model zanke

Morooka Hi, Matsumoto in Morooka Ha. [9] so se v nasprotju z drugimi avtorji geometrijskih modelov zanke v svojih študijah usmerili izključno v analizo krožnega enostavnega levo-desnega pletiva hlačnih nogavic, pri katerem zanka v obremenjenem – napetem stanju zavzema značilno raztegnjeno obliko; pri tem po trditvah avtorjev igelna glava s krakoma zavzema relativno majhen delež celotne dolžine zanke, medtem ko platinska glava zavzema večji delež. Čeprav Morookov in Matsumotov in Morookov model zanke [9] predstavlja zanko kot element na valju ležečega pletiva, tj. kot del prostorske strukture, gre za ploskovni model zanke, kar avtorji utemeljujejo z zanemarljivo debelino pletiva v primerjavi z dolžino igelne in platinske zanke pri pletivu za hlačne nogavice. Predpostavljena neraztezност preje [9] pomeni, da se pri obremenjevanju pletiva spremeni oblika zanke, tj. širina in višina zanke, pri čemer ostane dolžina zanke nespremenjena, saj gre za ploskovni model.

Morooka in Matsumoto in Morooka [9] so za splošno obliko zanke predpostavili *ohlapno strukturo pletiva*, pri kateri se igelni/platinski loki sosednjih zank ne stikajo niti v horizontalni niti v vertikalni smeri. Splošna enačba za izračun dolžine zanke (enačba 33) ob preimenovanju parametrov: dolžina zanke $\ell = YL / N_w$, širina zanke $A = 2\pi \cdot r / n_w$ in višina zanke $B = (d^2 + (h / n_c)^2)^{1/2}$ dobi obliko:

$$\ell = A + 2B + 4.28\, d_{pr} \qquad (37),$$

kar kaže odvisnost dolžine zanke od širine in višine zanke ter premera preje na podoben način kot pri Peirceovem modelu zanke ohlapnega pletiva (enačba 6) oz. pri Dalidovičevem splošnem modelu zanke (enačba 22). Koeficienti omenjenih treh modelov se razlikujejo, ker Dalidovič [6] predvideva polkrožno obliko igelne/platinske glave, Peirce [1] ravne odseke niti v polkrožni igelni/platinski glavi in krakih zanke, Morooka in Matsumoto in Morooka [9] pa ravne odseke niti le v igelni in platinski glavi zanke.

Morooka in Matsumoto in Morooka [9] so matematično opisali tudi parametre zanke pri ekstremnih pogojih, tj. pri maksimalnem horizontalnem, tj. prečnem oz. maksimalnem vertikalnem, tj. vzdolžnem raztegu pletiva. Ker preja po predpostavkah avtorjev ni raztezna, se premer preje in dolžina zanke pri raztezanju ne spremenita. Pri maksimalnem horizontalnem raztegu pride do *vertikalne normalne strukture*, tj. do stikanja igelnih in platinskih lokov sosednjih zančnih vrst, medtem ko je struktura pletiva v horizontalni smeri ohlapna, torej se sosednji zančni stolpci ne stikajo. Pri maksimalnem vertikalnem raztegu pletiva pride do *horizontalne normalne strukture*, tj. do stikanja sosednjih zančnih stolpcev, medtem ko je struktura pletiva v vertikalni smeri ohlapna, torej se igelni in platinski loki sosednjih zančnih vrst ne stikajo.

4 Sklepi

Geometrijski modeli zanke Peircea [1], Leafa in Glaskina [2], Mundena [3], Vekassyja [4], Suha [5], Dalidoviča [6, 7], Morooka in Matsumota in Marooka [9] temeljijo na predpostavkah, da ima preja konstantni premer, okrogel prerez ter je neraztezna in popolnoma upogljiva. Resnična, posebno sodobna pletiva so kompleksni materiali, ki ne ustrezajo uporabljenim idealiziranim predpostavkam.

Geometrijski modeli sicer eksplicitno ali implicitno predpostavljajo, da je geometrijska oblika zanke za vsak model konstantna, vendar nekateri pri posebnih pogojih predvidevajo:

- *spremembo premera preje* – Peirce [1] in Dalidovič [7] pri obremenjevanju pletiva,
- *spremembo oblike zanke* – Peirce [1] in Vekassy [4] pri spremembi ohlapnosti – zbitosti strukture pletiva, Suh [5] pri namakanju in pranju, Dalidovič [7], Korlinski [8] ter Morooko in Matsumoto in Morooko [9] pri obremenjevanju pletiva in
- *razteznost preje* – Dalidovič [7].

Geometrijski modeli so v okviru temeljnih predpostavk večinoma splošni: Leaf in Glaskin [2], Munden [3], Vekassy [4], Suh [5], Dalidovič [7], Korlinski [8], Morooka in Matsumoto in Morooka [9] ali pa opisujejo le strukturo, pri kateri se igelni in platinski loki sosednjih zank stikajo, tj. normalno strukturo pletiva, kot npr. Peirce [1]. Nekateri med njimi posamezno vrsto strukture predvidevajo kot poseben primer: Peirce [1] opisuje *ohlapno strukturo*, Vekassy [4] in Dalidovič [6] *normalno strukturo* ter Peirce [1] in Vekassy [4] *zbito strukturo*.

Za projektiranje in analizo parametrov votkovnega pletiva so najuporabnejši Peirceov, Dalidovičev in Mundenov model zanke; opisujejo normalno, ohlapno, pod omejenimi pogoji pa tudi zbito strukturo. Uporabni so predvsem za študij pletiv iz konvencionalnih prej. Zbito strukturo, ki je v sodobnosti značilna za pletiva z vgrajenim elastanskim jedrom, je mogoče opisati z Mundenovimi konstantami, ki imajo pomembno nižje vrednosti od tistih, ki opisujejo normalno do ohlapno strukturo. Vekassyjev model najbolje opisuje resnično, tridimenzionalno pleteno strukturo, a je za projektiranje pletiva na splošno preveč zapleten.

5 Viri

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